LevelSet Math Assessments:

Technical Guide

June 2021

Prepared by MetaMetrics for:

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Prepared by MetaMetrics, Inc. for Achieve3000, Inc. (https://www.achieve3000.com/) under Contract to Achieve3000, Inc. (Contract dated January 20, 2020).

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Introduction

Achieve3000, Inc. (achieve3000.com) delivers a comprehensive suite of digital solutions that are designed to accelerate literacy growth and deepen learning across the content areas (Achieve3000, Inc., 2020a). Using personalized and differentiated solutions, Achieve3000 provides equity for remote and on-site instruction, enabling educators to help all students achieve accelerated growth. For more than four million students in grades PreK-12, Achieve3000 improves high-stakes test performance and drives college and career readiness.

Launched in fall 2020, Achieve3000 Math offers a powerful experience to support math fluency and skills mastery. Similar to Achieve3000 Literacy, the solution delivers differentiated practice and intervention that closes knowledge gaps and builds confidence for all students, Kindergarten through Grade 12 (Achieve3000, Inc., 2020b). Achieve3000 Math (Achieve3000, Inc., n.d.) provides the following to differentiate mathematics instruction for all students:

- *LevelSet Math Assessments*: Assessments developed with MetaMetrics that determine students' individual Quantile measures so educators can tailor instruction and monitor growth. Three assessments (Beginning of Year, Middle of Year, and End of Year) are available at each grade level.
- *Aligned to Standards*: Comprehensive scope of state standards-aligned mathematics content from basic elementary fluency and numeracy to core high school topics in Algebra 1, Geometry, and Algebra 2.
- *Step-by-Step Scaffolding*: Dynamic math problems designed to move students step-by-step to find solutions, with a purposeful breakdown of the different pieces of knowledge needed.
- *Progressive Supports*: Supports increase if students struggle, first offering content that breaks down each problem into its component parts, then providing meaningful feedback on errors, and access to hints directly related to the problem, and finally video lessons on the topic.
- *Student Controls*: The ability to translate math problems into a language of choice and to have problems read aloud.

The Achieve3000 Math program for Grades K through 11/12 incorporate the *Quantile*® Framework for Mathematics. The Quantile Framework is a unique measurement system that uses a common scale and metric to assess a student's mathematical achievement level and the difficulty of specific skills and concepts. The Quantile Framework describes a student's ability to solve mathematical problems and the demand of the skills and concepts typically taught in kindergarten mathematics through algebra II, geometry, trigonometry, pre-calculus, and calculus. Quantile measures take the guesswork out of instruction by describing which mathematical skills and concepts the student has learned and is ready to learn. Quantile measures improve mathematics teaching and learning by targeting instruction and monitoring student growth toward proficiency standards and the mathematical demands of college and careers.

Periodically, students take the LevelSet Math assessments, which consist of mathematics test items aligned with the Common Core State Standards for Mathematics (National Governors Association & Council of Chief State School Officers, 2010a and 2010b). The results for Achieve3000 Math are reported on the Quantile scale. A developmental, or vertical, scale allows scores for students at different grade levels to be reported on a common scale. Since the Quantile scale is applied to both students and materials, it is possible to match students with mathematics materials of appropriate difficulty to facilitate improvement. Teachers and parents can use these Quantile measures to monitor student growth in mathematics achievement and forecast student readiness for instruction.

From winter 2019 through summer 2020, Achieve3000 developed the LevelSet Math assessments for Kindergarten through Grades 11/12, to be taken by all students using Achieve3000 Math. In addition, lessons were aligned with the Quantile Skills and Concepts (QSCs) and problem sets were calibrated to the Quantile scale using the Quantile Analyzer.

This technical guide for the LevelSet Math assessments provides users with a broad research foundation. Such a base is essential when deciding if and how the assessment results should be used and what kinds of inferences about a student's mathematical learning are permissible.

Background

In June of 2010, the National Governors Association Center for Best Practices and the Council of Chief State School Officers (CCSSO) released the Common Core State Standards (CCSS). These standards, developed for K-12 in English language arts and mathematics, establish clear goals for learning intended to prepare students for success in college and work. The mathematics standards outline challenging goals for student mathematical understanding and provide guidance on the skills and concepts that should be part of instruction at each grade level or course (Common Core State Standards for Mathematics, NGA Center and CCSSO, 2010a). The Quantile Skills and Concepts (QSCs) have been aligned with the Common Core State Standards for Mathematics.

In 2012, the National Governor's Association, Council of Chief State School Officers (CCSSO), Achieve, Council of Great City Schools, and the National Association of State School Boards of Education (NASBE) released the "K-8 Publisher's Criteria for the Common Core State Standards for Mathematics." In April 2013, the "High School Publisher's Criteria for the Common Core State Standards for Mathematics" were released. These documents were developed by the CCSSM writing team to provide criteria for materials aligned with the CCSSM and are based on the evidence-based designed principles of the CCSSM:

- *Focus*: focus strongly where the standards focus
- *Coherence*: think across grades, and link to major topics in each grade
- *Rigor*: in major topics, pursue with equal intensity: conceptual understanding, procedural skill and fluency, and applications.

Congress passed the No Child Left Behind (NCLB) Act of 2001, a reauthorization of the Elementary and Secondary Education Act (ESEA). This act required states to administer annual assessments to all students in Grades 3 through 8 and once in high school by the end of the 2005-2006 school year. In 2015, the ESEA was again reauthorized with the signing of the Every Student Succeeds Act (ESSA) by President Obama on December 10th (Klein, 2015a, 2015b). Under the ESSA legislation, states will select and design tests of their choosing, but the tests are to be aligned with the respective state's mathematics standards. This legislation requires states to:

- Adopt “challenging” academic standards for reading and mathematics.
- Create statewide proficiency standards for student achievement in reading and mathematics in Grades 3 through 8 and in high school.
- Define these standards according to student performance on statewide outcome assessments and break out the data for whole schools and subgroups of students (English-learners, students in special education, racial minorities, and those in poverty).
- (and also at the district level) Use locally-developed, evidence-based interventions, in the bottom 5% of schools and in schools where less than two-thirds of the students graduate.

Although many states have made gains in mathematics achievement, nationally, students still have much room for progress, as seen in the 2015 National Assessment of Educational Progress (NAEP) results for mathematics (National Center for Education Statistics, 2015). At the fourth grade, six out of ten (60%) students performed at the Basic level or below and about four out of ten (40%) students performed at or above Proficient. Only 7% performed at the Advanced level. At the eighth grade, two thirds (67%) of the students performed at the Basic level or below, one third (33%) performed at or above Proficient, and 8% performed at the Advanced level.

The LevelSet Math assessments were developed using the Quantile Framework which has been aligned with the CCSS (considered “challenging” academic standards under ESSA) and achieves the “focus” principle. The Knowledge Clusters and the Quantile Teacher Assistant (https://math-tools.quantiles.com/quantile-teacher-assistant/) of the Quantile Framework and the customizable content of the CCSS curriculum can be utilized to achieve the “coherence” principle. And, the items have been developed to achieve the “rigor” principle.

Features of LevelSet Math Assessments

The LevelSet Math assessments are research-based, scientifically valid, and reliable. Several specific features of the LevelSet Math assessments are noteworthy.

- Items were selected from the Quantile Item Bank and are aligned with materials typically covered in Kindergarten through high school.

- The LevelSet Math assessments are linked with the Quantile scale and, as such, the item calibrations used to convert a raw score (number correct) into the Quantile metric are provided by the Quantile Theory. The calibration methods used to calibrate the test forms and the test items are the same methods used to measure lessons and skills. Thus, students and mathematics skills and lessons are all placed on the same metric.

- The LevelSet Math assessments are appropriate for individual, small group, and large group administration settings.

- The test format supports quick administration in an un-timed, low-pressure format.

- No extensive or specialized preparation is needed to administer the LevelSet Math assessments, although proper interpretation and use of the results requires an understanding of The Quantile Framework for Mathematics.

- The LevelSet Math assessments support rapid objective scoring by computer.

- The LevelSet Math assessments use a Bayesian scoring algorithm which incorporates past performance to predict future performance. Bayesian methodology provides a paradigm for combining prior information with current data, both subject to uncertainty, to calculate an estimate of current status, which is again subject to uncertainty. This methodology connects the administration of each assessment, and thus it produces more precise measurements when compared with independent assessments.

Purposes and Uses of the LevelSet Math Assessments

The LevelSet Math assessments are designed to measure a student's readiness for instruction on appropriate grade-level material as defined by the Common Core State Standards for Mathematics (CCSS). The results of the LevelSet Math assessments can be used to measure where students stand in the development of their overall mathematical ability.

One outcome of the LevelSet Math assessments is the location of the student on the Quantile Map (Appendix A). After a student's mathematical achievement is measured, it is possible to forecast how well the student will understand instruction on other mathematical skills that are measured on the Quantile scale. In other words, students and skills are measured using the same Quantile scale making it possible to directly compare a student's ability and his or her understanding of a specific skill. When a student and a Quantile measure match, the Quantile Framework forecasts 50% understanding, which describes the situation in which the student is ready for instruction on the topic or skill. The operational definition of 50% understanding is that if given 100 items from a first night's homework prior to an introductory lesson, the student should be able to correctly answer 50 of those items.

When a grade-level appropriate skill has a Quantile measure of 200Q higher than the student measure, the Quantile Framework forecasts 25% understanding. When the student measure exceeds the skill demand measure by 200Q, the forecasted understanding is 75%. The challenge for each educator is to identify the skills and concepts that each student is ready to be instructed on at the right time. The Quantile Framework provides educators with information related to the observed difficulty of skills and concepts and the content progressions (Quantile Knowledge Clusters). Examples of concepts and skills at various points on the Quantile scale are shown on the Quantile Map (see Appendix A).

The LevelSet Math assessment results inform the use of instructional products and programs to support school districts' efforts to accelerate the learning path of both struggling students and students who are successful in mathematics instruction. State educational agencies (SEAs), local educational agencies (LEAs), and schools can use Title 1, Part A funds associated with the American Recovery and Reinvestment Act of 2009 (ARRA) to identify, create, and structure opportunities and strategies to strengthen education, drive school reform, and improve the academic achievement of at-risk students using funds under Title I, Part A of the Elementary and Secondary Education Act of 1965 (ESEA) (U.S. Department of Education, 2009). Educators can use tiered intervention strategies to provide support for students who are “at risk” of not meeting state performance levels that define “proficient” achievement. One such tiered approach is Response to Intervention (RTI), which involves providing the most appropriate instruction, services, and scientifically based interventions to struggling students—with increasing intensity at each tier of instruction (Cortiella, 2005).

The LevelSet Math assessments are academic assessments that can be used to assess the placement of students in Tier 2 and Tier 3 interventions (targeted interventions and progress monitoring of students who have not been successful in Tier 1). At Tier 2, students who are struggling can receive targeted intervention informed by the LevelSet Math assessment results. Students who continue to struggle can receive additional, intensive intervention at Tier 3 and may be referred to special education to ensure that instruction meets their specific needs. Regular monitoring of students’ responses to instruction is important in determining whether a student should move from one level to another.

Development Groups

Achieve3000, Inc. provided the vision and guided the development of the content specifications of the LevelSet Math assessments. Achieve3000, Inc. approved the final item sets, implemented the scoring and reporting protocols, and implemented the production of the LevelSet Math assessments. Achieve3000, Inc. also integrated the Quantile calibrations of the problems sets and the results from the Quantile Sequencing Service to aid in connecting students with appropriate materials for differentiated and personalized instruction with challenge but not frustration.

MetaMetrics, Inc. managed the overall development of the assessments by designing the assessments, developing the test items, coordinating the test development, and designing the scoring and reporting protocols. In addition, MetaMetrics calibrated the problem sets using the Quantile Analyzer and sequenced them with the Quantile Sequencing Service.

Limitations of LevelSet Math Assessments

The LevelSet Math assessments can provide useful information for matching resources and students. As with any other assessment, results from this test are just one source of evidence about a student’s level of mathematical understating. Instructional decisions are best made when using multiple sources of evidence about a student. Other sources include standardized test data, instructional group placement, lists of grade-level materials and objectives, and, most importantly, teacher judgment. *One measure of student performance, taken on one day, is never sufficient to make high-stakes student-level decisions such as summer school placement or retention.*

The Quantile Framework for Mathematics provides a common metric for combining different sources of information about a student into a best overall judgment of the student’s ability expressed in the Quantile metric. Achieve3000, Inc. and MetaMetrics encourage users of Achieve3000 Math to employ multiple measures when deciding where to locate a student on the Quantile scale.

The Quantile Framework for Mathematics

The Quantile Framework is a scale that describes a student's mathematical achievement. Similar to how degrees on a thermometer measure temperature, the Quantile Framework uses a common metric—the Quantile—to scientifically measure a student's ability to reason mathematically, monitor a student's readiness for mathematics instruction, and locate a student on its taxonomy of mathematical skills, concepts, and applications.

The Quantile Framework uses this common metric to measure many different aspects of education in mathematics. The same metric can be applied to measure the materials used in instruction, to calibrate the assessments used to monitor instruction, and to interpret the results that are derived from the assessments. The result is an anchor to which resources, concepts, skills, and assessments can be connected.

There are dozens of mathematics tests that measure a common construct and report results in proprietary, nonexchangeable metrics. Not only are all of the tests using different units of measurement, but all use different scales on which to make measurements. Consequently, it is difficult to connect the test results with materials used in the classroom. The alignment of materials and linking of assessments with the Quantile Framework enables educators, parents, and students to communicate and improve mathematics learning. The benefits of having a common metric include being able to:

(1) Develop individual multiyear growth trajectories that denote a developmental continuum from the early elementary level to Algebra II and Pre-calculus. The Quantile scale is vertically constructed, so the meaning of a Quantile measure is the same regardless of grade level.
(2) Monitor and report student growth that meets the needs of state accountability systems.
(3) Help classroom teachers make day-to-day instructional decisions that foster acceleration and growth toward algebra readiness and through the next several years of secondary mathematics.

Student learning is facilitated when instruction and practice are tailored to the individual's abilities (Vygotsky, 1978). For example, when mathematics instruction is differentiated and targeted to individual students' mathematical abilities, student achievement and growth are improved (Clements, Sarama, Spitler, Lange, & Wolfe, 2011; Gavin, Casa, Adelson, Carroll, & Sheffield, 2009; Poncy, Fontenelle, & Skinner, 2013; Matthews, Ritchotte, & McBee, 2013; McCoach, Gubbins, Foreman, Rubenstein, & Rambo-Hernandez, 2014; Tieso, 2005). Because differentiation and targeting optimize the balance between challenge and success, students can experience the efficacy of their own learning efforts, thereby reinforcing a growth mindset (Dweck, 2008). Furthermore, aligning the cognitive demands of learning materials with students' growth trajectories facilitates student growth and may help students set personal growth goals as well (Martin, 2015; Mok, McInerney, Zhu & Or, 2014).

Accordingly, the Quantile Framework Theory of Action posits that using the Quantile Framework to optimally differentiate learning and instruction improves student ability and growth and also contributes to a growth mindset.

In order to develop the Quantile Framework, the following tasks were undertaken:

(1) The development of a structure of mathematics that spans the developmental continuum from first grade content through Algebra I, Geometry, and Algebra II content.
(2) The production of a bank of items that have been field tested.
(3) The development of the Quantile scale (multiplier and anchor point) based on the calibrations of the field-test items.
(4) The validation of the measurement of mathematics ability as defined by the Quantile Framework.

Each of these tasks is described in the sections that follow.

Structure of the Quantile Framework

In order to develop a framework of mathematical ability, first a structure needs to be established. The structure of the Quantile Framework is organized around two principles—(1) mathematics and mathematical ability are developmental in nature and (2) mathematics is a content area.

The Common Core State Standards for Mathematics describe one of the key shifts in mathematics called for – rigor. Rigor is defined as the pursuit of "conceptual understanding, procedural skills and fluency, and application with equal intensity" (National Governor's Association and Council of Chief State School Officers, 2014).

- *Conceptual understanding:* The standards call for conceptual understanding of key concepts, such as place value and ratios. Students must be able to access concepts from a number of perspectives in order to see math as more than a set of mnemonics or discrete procedures.

- *Procedural skills and fluency:* The standards call for speed and accuracy in calculation. Students must practice core functions, such as single-digit multiplication, in order to have access to more complex concepts and procedures. Fluency must be addressed in the classroom or through supporting materials, as some students might require more practice than others.

- *Application:* The standards call for students to use math in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.

When developing the Quantile Framework, MetaMetrics recognized that in order to adequately address the scope and complexity of mathematics, multiple proficiencies and competencies must be assessed. The Quantile Framework is an effort to recognize and define a developmental context of mathematics instruction. This notion is consistent with the National Council of Teachers of Mathematics' (NCTM) conclusions about the importance of school mathematics for college and career readiness presented in the *Administrator's Guide: Interpreting the Common Core State Standards to Improve Mathematics Education*, published in 2011.

Mathematical Strands

A strand is a major subdivision of mathematical content. The strands describe what students should know and be able to do. The National Council of Teachers of Mathematics (NCTM) publication Principles and Standards for School Mathematics (2000, hereafter NCTM Standards) outlined ten standards—five content standards and five process standards. These content standards are Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. The process standards are Communications, Connections, Problem Solving, Reasoning, and Representation.

As of March 2014, the Common Core State Standards for Mathematics (CCSSM) were adopted in 44 states, the Department of Defense Education Activity, Washington D.C., Guam, the Northern Mariana Islands and the U.S. Virgin Islands. The CCSSM identify critical areas of mathematics that students are expected to learn each year from kindergarten through grade 8. The critical areas are divided into domains which differ at each grade level and include Counting and Cardinality, Operations and Algebraic Thinking, Number and Operations in Base Ten, Number and Operations-Fractions, Ratios and Proportional Relationships, The Number System, Expressions and Equations, Functions, Measurement and Data, Statistics and Probability, and Geometry. The CCSSM for grades 9–12 are organized by six conceptual categories: Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability.

The six strands of the Quantile Framework bridge the Content Standards of the NCTM Standards and the domains specified in the CCSSM.

- *Algebra and Algebraic Thinking.* The use of symbols and variables to describe the relationships between different quantities is covered by algebra. By representing unknowns and understanding the meaning of equality, students develop the ability to use algebraic thinking to make generalizations. Algebraic representations can also allow the modeling of an evolving relationship between two or more variables.

- *Number Sense.* Students with number sense are able to understand a number as a specific amount, a product of factors, and the sum of place values in expanded form. These students have an in-depth understanding of the base-ten system and understand the different representations of numbers.

- *Numerical Operations.* Students perform operations using strategies and standard algorithms on different types of numbers but can also use estimation to simplify computation and to determine how reasonable their results are. This strand also encompasses computational fluency.

- *Measurement.* The description of the characteristics of an object using numerical attributes is covered by measurement. The strand includes using the concept of a unit to determine length, area, and volume in the various systems of measurement, and the relationship between units of measurement within and between these systems.

- *Geometry*. The characteristics, properties, and comparison of shapes and structures are covered by geometry, including the composition and decomposition of shapes. Not only does geometry cover abstract shapes and concepts, but it provides a structure that can be used to observe the world.

- *Data Analysis, Statistics, and Probability*. The gathering of data and interpretation of data are included in data analysis, probability, and statistics. The ability to apply knowledge gathered using mathematical methods to draw logical conclusions is an essential skill addressed in this strand.

The Quantile Skill and Concept

Within the Quantile Framework, a Quantile Skill and Concept, or QSC, describes a specific mathematical skill or concept a student can acquire. These QSCs were arranged in an orderly progression to create a taxonomy called the Quantile scale. Examples of QSCs include:

- Know and use addition and subtraction facts to 10 and understand the meaning of equality.
- Use addition and subtraction to find unknown measures of non-overlapping angles.
- Determine the effects of changes in slope and/or intercepts on graphs and equations of lines.

The QSCs used within the Quantile Framework were developed during Spring 2003, for Grades 1 through 8, Grade 9 (Algebra I) and Grade 10 (Geometry). The framework was extended to Algebra II and revised during Summer/Fall 2003. The content was finally extended to include material typically taught in Kindergarten and Grade 12 (Pre-calculus) during the Summer/Fall, 2007. And in 2019, the framework was once again extended to include Statistics and Calculus.

The first step in developing a content taxonomy was to review the curricular frameworks from a variety of sources (e.g., National Council of Teachers of Mathematics (NCTM), National Assessment of Educational Progress: 2005 Pre-Publication Edition, North Carolina, California, Florida, Illinois, and Texas). The review of the content frameworks resulted in the development of a list of QSCs spanning the content typically taught in kindergarten through Algebra I, geometry, and Algebra II. Each QSC consists of a description of the content, a unique identification number, the grade at which it typically first appears, and the strand with which it is associated.

The Quantile Framework Map (Appendix A) presents a picture of the construct of mathematics ability. The map is organized by the six strands and describes the development of mathematics from basic skills to sophisticated problem solving. Exemplar QSCs and problems are used to annotate the Quantile scale and the strands. QSCs are located on the Quantile scale at the point corresponding to the mean of the ensemble of items addressing that QSC from two large, national studies (Quantile Framework field study and *PASeries* Math field study described later in this document and the aforementioned 2019 Quantile extension to include Statistics and Calculus) and from additional field studies as new QSCs are proposed and investigated. Items are located on the Quantile scale corresponding to their Quantile measure based on the Quantile Framework field study.

Quantile Scale Development

The second step in the process of developing the Quantile Framework of Mathematics was to develop and field test a bank of items that could be used in future linking studies. Item bank development for the Quantile Framework went through several stages—content specification, item writing and review, field-testing and analyses, and final evaluation.

Item Specification and Development

Each QSC developed during the design of the Quantile Framework was paired with a particular strand and identified as typically being taught at a particular grade level. The curricular frameworks from Florida, North Carolina, Texas, and California were synthesized to identify the QSCs instructed and/or assessed at each grade level. If a QSC was included in any state framework it was included in the list of QSCs for which items were to be developed for use with the Quantile Framework field study.

During the summer and fall of 2003, over 1,400 items were developed to assess the QSCs associated with content in grades 1 through Algebra II. The items were written and reviewed by mathematics educators trained to develop multiple-choice items (Haladyna, 1994). Each item was associated with a strand and a QSC. In the development of the Quantile Framework item bank, the reading demand of the items was kept as low as possible to ensure that the items were testing mathematics achievement and not reading. Additional Statistics and Calculus items were developed and field tested in 2019 and are included in the Quantile Item Bank.

Item Writing and Review

Item writers were experienced teachers and item-development specialists who had experience with the everyday mathematical ability of students at various levels. The use of individuals with these types of experiences helped to ensure that the items were valid measures of mathematics. Item writers were provided with training materials concerning the development of multiple-choice items and the Quantile Framework. The item writing materials also contained incorrect and ineffective items that illustrated the criteria used to evaluate items and corrections based on those criteria. The final phase of item writer training was a short practice session with three items.

Item writers were also given additional training related to sensitivity issues. Part of the item writing materials address these issues and identify areas to avoid when developing items. These materials were developed based on material published on universal design and fair access—equal treatment of the sexes, fair representation of minority groups, and the fair representation of disabled individuals.

Items were reviewed and edited by a group of specialists that represented various perspectives—test developers, editors, and curriculum specialists. These individuals examined each item for sensitivity issues and for the quality of the response options. During the second stage of the item review process, items were approved, approved with edits, or deleted.

Linking and Field-Test Design

The next stage in the development of the Quantile item bank was the field-testing of all of the items. First, individual test items were compiled into leveled assessments distributed to groups of students. The data gathered from these assessments were then analyzed using a variety of statistical methods. The final result was a bank of test items appropriately placed within the Quantile scale, suitable for determining the mathematical achievement of students on this scale. Assessment forms were developed for 10 levels for the purposes of field-testing. Levels 2 through 8 were aligned with the typical content taught in grades 2 through 8, Level 9 was aligned with the typical content taught in Algebra I, Level 10 was aligned with the typical content taught in Geometry, and Level 11 was aligned with the typical content taught in Algebra II. For each level, three forms were developed with each form containing 30 items.

The final field tests were composed of 685 unique items. Besides the 660 items mentioned above, two sets of 12 linking items were developed to serve as below-level items for Grade 2 and above-level items for Algebra II. Two additional Algebra II items were developed to ensure coverage of all the QSCs at that level.

Linking the test levels vertically (across grades) employed a common-item test design (design in which items are used on multiple forms). In this design, multiple tests are given to nonrandom groups, and a set of common items is included in the test administration to allow some statistical adjustments for possible sample-selection bias. This design is most advantageous where the number of items to be tested (treatments) is large and the consideration of cost (in terms of time) forces the experiment to be smaller than is desired (Cochran and Cox, 1957).

Quantile Framework Field Study and Analysis

The Quantile Framework field study was conducted in February 2004. Thirty-seven schools from 14 districts across six states (California, Indiana, Massachusetts, North Carolina, Utah, and Wisconsin) agreed to participate in the study. Data were received from 34 of the schools (two elementary and one middle-school did not return data). A total of 9,847 students in grades 2 through 12 were tested. The number of students per school ranged from 74 to 920. The schools were diverse in terms of geographic location, size, and type of community (e.g., suburban; small town, city, or rural communities; and urban). See *Table 1* for information about the sample at each grade level and the total sample. See *Table 2* for test administration forms by level.

Table 1. Field-study participation by grade and gender.

Grade Level	*N*	Percent Female (*N*)	Percent Male (*N*)
2	1,283	48.1 (562)	51.9 (606)
3	1,354	51.9 (667)	48.1 (617)
4	1,454	47.7 (644)	52.3 (705)
5	1,344	48.9 (622)	51.1 (650)
6	976	47.7 (423	52.3 (463)
7	1,250	49.8 (618)	50.2 (622)
8	1,015	51.9 (518)	48.1 (481)
9	489	52.0 (252)	48.0 (233)
10	259	48.6 (125)	51.4 (132)
11	206	49.3 (101)	50.7 (104)
12	143	51.7 (74)	48.3 (69)
Missing	74	39.1 (9)	60.9 (14)
Total	9,847	49.6 (4,615)	50.4 (4,696)

Table 2. Test-form administration by level.

Test Level	*N*	Missing	Form 1	Form 2	Form 3
2	1,283	4	453	430	397
3	1,354	7	561	387	399
4	1,454	17	616	419	402
5	1,344	3	470	448	423
6	917	13	322	293	289
7	1,309	6	463	429	411
8	1,181	16	387	391	387
9	415	4	141	136	134
10	226	5	73	77	71
11	313	10	102	101	100
Missing	51	31	9	8	3
Total	9,847	116	3,596	3,119	3,016

Students given Levels 2 through 11 were provided with rulers and students given Levels 3 through 11 were provided with protractors. For students given Levels 5 through 8 and 10 and 11, formulas were provided on the back of the test booklet. Administration time was approximately 45 minutes at each level. Students given Level 2 could have the test read aloud and mark in the test booklet if that was typical of instruction.

Field-Test Analyses. At the conclusion of the field test, complete data was available for 9,678 students. Data were deleted if test level or test form was not indicated or the answer sheet was blank. The field-test data were analyzed using both the classical measurement model and the Rasch (one-parameter logistic item response theory) model. Item statistics and descriptive information (item number, field test form and item number, QSC, and answer key) were printed for each item and attached to the item record. The item record contained the statistical, descriptive, and historical information for an item; a copy of the item itself as it was field-tested; any comments by reviewers; and the psychometric notations. Each item had a separate item record.

Field-Test Analyses—Classical Measurement. For each item, the *p*-value (percent correct) and the point-biserial correlation between the item score (correct response) and the total test score were computed. Point-biserial correlations were also computed between each of the incorrect responses and the total score. In addition, frequency distributions of the response choices (including omits) were tabulated (both actual counts and percents). Items with point-biserial correlations less than 0.10 were removed from the item bank. *Table 3* displays the summary item statistics.

Table 3. Summary item statistics from the Quantile Framework field study (February 2004).

Level	**Number of Items Tested**	***p*-value Mean (Range)**	**Correct Response Point-Biserial Correlation Mean (Range)**	**Incorrect Responses Point-Biserial Correlation Mean (Range)**
2	90	0.583 (0.12 – 0.95)	0.322 (-0.15 – 0.56)	-0.209 (-0.43 – 0.12)
3	90	0.532 (0.11 – 0.93)	0.256 (-0.08 – 0.52)	-0.221 (-0.54 – 0.02)
4	90	0.552 (0.12 – 0.92)	0.242 (-0.21 – 0.50)	-0.222 (-0.48 – 0.12)
5	90	0.535 (0.12 – 0.95)	0.279 (-0.05 – 0.50)	-0.225 (-0.45 – 0.05)
6	90	0.515 (0.04 – 0.86)	0.244 (-0.08 – 0.45)	-0.218 (-0.46 – 0.09)
7	90	0.438 (0.10 – 0.77)	0.294 (-0.12 – 0.56)	-0.207 (-0.46 – 0.25)
8	90	0.433 (0.10 - 0.81)	0.257 (-0.15 – 0.50)	-0.201 (-0.45 – 0.13)
9	90	0.396 (0.10 – 0.79)	0.208 (-0.19 – 0.52)	-0.193 (-0.53 – 0.22)
10	88	0.511 (0.01 – 0.97)	0.193 (-0.26 – 0.53)	-0.205 (-0.55 – 0.18)
11	90	0.527 (0.09 – 0.98)	0.255 (-0.09 – 0.51)	-0.223 (-0.52 – 0.07)

Field-Test Analyses—Bias. Differential item functioning (DIF) examines the relationship between the score on an item and group membership while controlling for ability. The Mantel-Haenszel procedure has become "the most widely used methodology [to examine differential item functioning] and is recognized as the testing industry standard" (Roussos, Schnipke, and Pashley, 1999, p. 293). The Mantel-Haenszel procedure examines DIF by examining $j \times 2$ contingency tables, where j is the number of different levels of ability actually achieved by the examinees (actual total scores received on the test). The focal group is the group of interest and the reference group serves as a basis for comparison for the focal group (Dorans and Holland, 1993; Camilli and Shepherd, 1994).

The Mantel-Haenszel chi-square statistic tests the alternative hypothesis that there is a linear association between the row variable (score on the item) and the column variable (group membership). The χ^2 distribution has 1 degree of freedom and is determined as

$$Q_{MH} = (n-1)r^2 \qquad \text{(Equation 1)}$$

where *r* is the Pearson correlation between the row variable and the column variable (SAS Institute, 1985).

The Mantel-Haenszel (MH) Log Odds Ratio statistic is used to determine the direction of differential item functioning (SAS Institute Inc., 1985). This measure is obtained by combining the odds ratios, α_j, across levels with the formula for weighted averages (Camilli and Shepherd, 1994, p. 110):

$$\alpha_j = \frac{p_{Rj} / q_{Rj}}{p_{Fj} / q_{Fj}} = \frac{\Omega_{Rj}}{\Omega_{Fj}} \qquad \text{(Equation 2)}$$

For this statistic, the null hypothesis of no relationship between score and group membership, or that the odds of getting the item correct are equal for the two groups, is not rejected when the odds ratio equals 1. For odds ratios greater than 1, the interpretation is that an individual at score level *j* of the Reference Group has a greater chance of answering the item correctly than an individual at score level *j* of the Focal Group. Conversely, for odds ratios less than 1, the interpretation is that an individual at score level *j* of the Focal Group has a greater chance of answering the item correctly than an individual at score level *j* of the Reference Group. The Breslow-Day Test is used to test whether the odds ratios from the *j* levels of the score are all equal. When the null hypothesis is true, the statistic is distributed approximately as a χ^2 with *j*-1 degrees of freedom (Camilli and Shepherd, 1994; SAS Institute, 1985).

For the gender analyses, males (approximately 50.4% of the population) were defined as the reference group and females (approximately 49.6% of the population) were defined as the focal group.

The results from the Quantile Framework linking study were reviewed for inclusion on linking studies. The following statistics were reviewed for each item: *p*-value, point-biserial correlation, and DIF estimates. Items that exhibited extreme statistics were removed from the item bank (47 out of 685).

From the studies conducted with the Quantile Framework item bank (Palm Beach County [FL] linking study, Mississippi linking study, DoDEA/TerraNova linking study, and Wyoming linking study), approximately 6.9% of the items in any one study are flagged as exhibiting DIF using the Mantel-Haenszel statistic and the *t*-statistic from Winsteps. For each linking study the following steps are used to review the items: (1) flag items exhibiting DIF, (2) review items to determine if the content of the item is something that we want all students to know and be able to do, and (3) make decision to retain or delete items.

Field-Test Analyses—Rasch Item Response Theory. Classical test theory has two basic shortcomings: (1) the use of item indices whose values depend on the particular group of examinees from which they were obtained, and (2) the use of examinee ability estimates that depend on the particular choice of items selected for a test. The basic premises of item response theory (IRT) overcome these shortcomings by predicting the performance of an examinee on a test item based on a set of underlying abilities (Hambleton and Swaminathan, 1985). The relationship between an examinee's item performance and the set of traits underlying item performance can be described by a monotonically increasing function called an item characteristic curve (ICC). This function specifies that as the level of the trait increases, the probability of a correct response to an item increases.

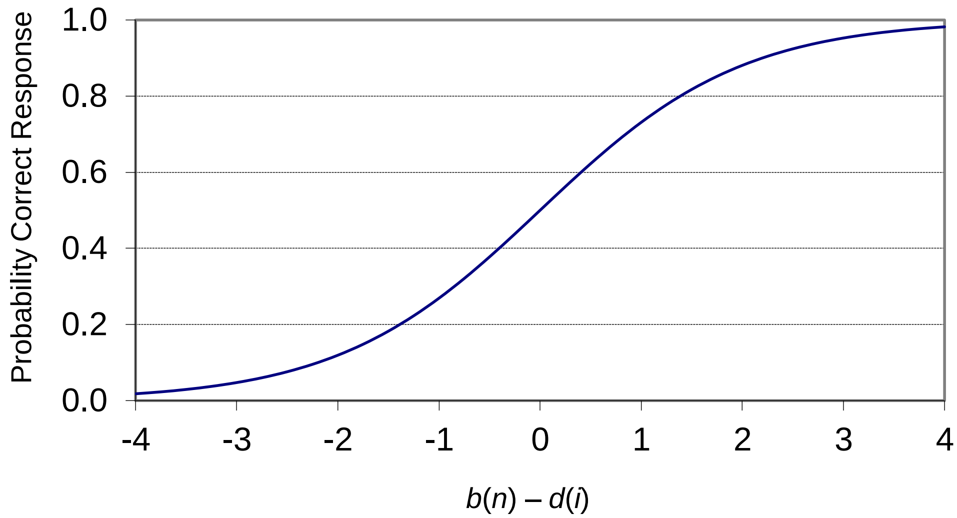
The conversion of observations into measures can be accomplished using the Rasch (1980) model, which states a requirement for the way that item calibrations and observations (count of correct items) interact in a probability model to produce measures. The Rasch IRT model expresses the probability that a person (*n*) answers a certain item (*i*) correctly by the following relationship:

$$P_{ni} = \frac{e^{b_n - d_i}}{1 + e^{b_n - d_i}} \qquad \text{(Equation 3)}$$

where d_i is the difficulty of item i ($i = 1, 2, \ldots,$ number of items);
b_n is the ability of person n ($n = 1, 2, \ldots,$ number of persons);
$b_n - d_i$ is the difference between the ability of person n and the difficulty of item i; and
P_{ni} is the probability that examinee n responds correctly to item i
(Hambleton and Swaminathan, 1985; Wright and Linacre, 1994).

This measurement model assumes that item difficulty is the only item characteristic that influences the examinee's performance such that all items are equally discriminating in their ability to identify low-achieving persons and high achieving persons (Bond and Fox, 2001; and Hambleton, Swaminathan, and Rogers, 1991). In addition, the lower asymptote is zero, which specifies that examinees of very low ability have zero probability of correctly answering the item. The Rasch model has the following assumptions: (1) unidimensionality—only one ability is assessed by the set of items; and (2) local independence—when abilities influencing test performance are held constant, an examinee's responses to any pair of items are statistically independent (conditional independence, i.e., the only reason an examinee scores similarly on several items is because of his or her ability, not because the items are correlated). The Rasch model is based on fairly restrictive assumptions, but it is appropriate for criterion-referenced assessments. *Figure 1* graphically shows the probability that a person will respond correctly to an item as a function of the difference between a person's ability and an item's difficulty.

Figure 1. The Rasch Model—the probability person n responds correctly to item i.



An assumption of the Rasch model is that the probability of a response to an item is governed by the difference between the item calibration (d_i) and the person's measure (b_n). From an examination of the graph in *Figure 1*, when the ability of the person matches the difficulty of the item ($b_n – d_i = 0$), then the person has a 50% probability of responding to the item correctly.

The number of correct responses for a person is the probability of a correct response summed over the number of items. When the measure of a person greatly exceeds the calibration (difficulties) of the items ($b_n – d_i > 0$), then the expected probabilities will be high and the sum of these probabilities will yield an expectation of a high "number correct." Conversely, when the item calibrations generally exceed the person measure ($b_n – d_i < 0$), the modeled probabilities of a correct response will be low and the expectation will be a low "number correct."

Thus, Equation 3 can be rewritten in terms of the number of correct responses of a person on a test

$$O_p = \sum_{i=1}^{L} \frac{e^{b_n - d_i}}{1 + e^{b_n - d_i}} \qquad \text{(Equation 4)}$$

where O_p is the number of correct responses of person p and L is the number of items on the test.

When the sum of the correct responses and the item calibrations (d_i) is known, an iterative procedure can be used to find the person measure (b_n) that will make the sum of the modeled probabilities most similar to the number of correct responses. One of the key features of the Rasch IRT model is its ability to place both persons and items on the same scale. It is possible to predict the odds of two individuals being successful on an item based on knowledge of the relationship between the abilities of the two individuals. If one person has an ability measure that is twice as high as that of another person (as measured by *b*—the ability scale), then he or she has twice the odds of successfully answering the item.

Equation 4 possesses several distinguishing characteristics:

- The key terms from the definition of measurement are placed in a precise relationship to one another.
- The individual responses of a person to each item on an instrument are absent from the equation. The only information that appears is the "count correct" (O_p), thus confirming that the raw score (i.e., number of correct responses) is "sufficient" for estimating the measure.
- For any set of items the possible raw scores are known. When it is possible to know the item calibrations (either theoretically or empirically from field studies), the only parameter that must be estimated in Equation 4 is the person measure that corresponds to each observable count correct. Thus, when the calibrations (d_i) are known, a correspondence table linking observation and measure can be constructed without reference to data on other individuals.

All students and items were submitted to a Winsteps analysis using a logit convergence criterion of 0.0001 and a residual convergence criterion of 0.001. Items that a student skipped were treated as missing, rather than being treated as incorrect. Only students who responded to at least 20 items were included in the analyses (22 students were omitted, 0.22%). The Quantile measure comes from multiplying the logit value by 180 and is anchored at 656Q. The multiplier and the anchor point will be discussed in a later section. *Table 4* shows the mean and median Quantile measures for all students with complete data at each grade level. While there is not a monotonically increasing trend in the mean and median Quantile measures in Grades 6 and 7, the measures are not significantly different. Results from other studies (e.g., *PASeries* Math described beginning on page 25 exhibit a monotonically increasing function.

Table 4. Mean and median Quantile measure for students with complete data (N = 9,656).

Grade Level	***N***	**Quantile measure Mean (SD)**	**Quantile measure Median**
2	1,275	320.68 (189.11)	323
3	1,339	511.41 (157.69)	516
4	1,427	655.45 (157.50)	667
5	1,337	790.06 (167.71)	771
6	959	871.82 (153.02)	865
7	1,244	860.52 (174.16)	841
8	1,004	929.01 (157.63)	910
9	482	958.69 (152.81)	953
10	251	1019.97 (162.87)	1005
11	200	1127.34 (178.57)	1131
12	138	1185.90 (189.19)	1164

Figure 2 shows the relationship between grade level and Quantile measure. The following box-and-whisker plots (*Figures 2, 3* and *4*) show the progression of the *y*-axis scores from grade to grade (the *x*-axis). For each grade, the box refers to the interquartile range. The line within the box indicates the median and the + indicates the mean. The end of each whisker shows the minimum

and maximum values of the *y*-axis which is the Quantile measure. Across all students, the correlation between grade and Quantile measure was 0.76.

All students with outfit mean square statistics greater than or equal to 1.8 were removed from further analyses. A total of 480 students (4.97%) were removed from further analyses. The number of students removed ranged from 8.47% (108) in grade 2 to 2.29% (22) in grade 6 with a mean percent decrease of 4.45% per grade.

All remaining students (9,176) and all items were submitted to a Winsteps analysis using a logit convergence criterion of 0.0001 and a residual convergence criterion of 0.001. Items that a student skipped were treated as missing, rather than being treated as incorrect. Only students who responded to at least 20 items were included in the analyses. *Table 5* shows the mean and median Quantile measures for the final set of students at each grade level. *Figure 3* shows the results from the final set of students. The correlation between grade level and Quantile measure is 0.78.

Figure 2. Box-and-whisker plot of the Rasch ability estimates of all students with complete data (N = 9,656).

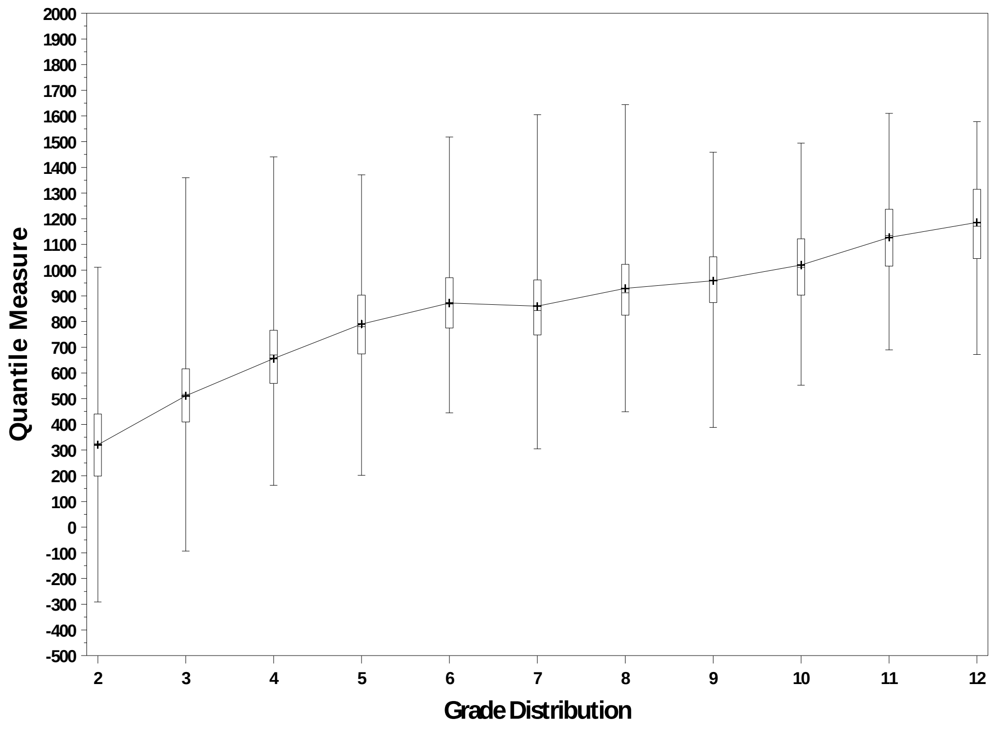


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Table 5. Mean and median Quantile measure for the final set of students (N = 9,176).

Grade Level	*N*	Logit Value Median	Quantile measure Mean (Median)
2	1,167	-2.800	289.03 (292)
3	1,260	-1.650	502.18 (499)
4	1,352	-0.780	652.60 (656)
5	1,289	0.000	795.25 (796)
6	937	0.430	880.77 (874)
7	1,181	0.370	877.75 (863)
8	955	0.810	951.41 (942)
9	466	1.020	982.62 (980)
10	244	1.400	1044.08 (1048)
11	191	2.070	1160.49 (1169)
12	134	2.295	1219.87 (1210)

Figure 3. Box-and-whisker plots of the Rasch ability estimates for the final sample of students with outfit statistics less than 1.8 (N = 9,176).

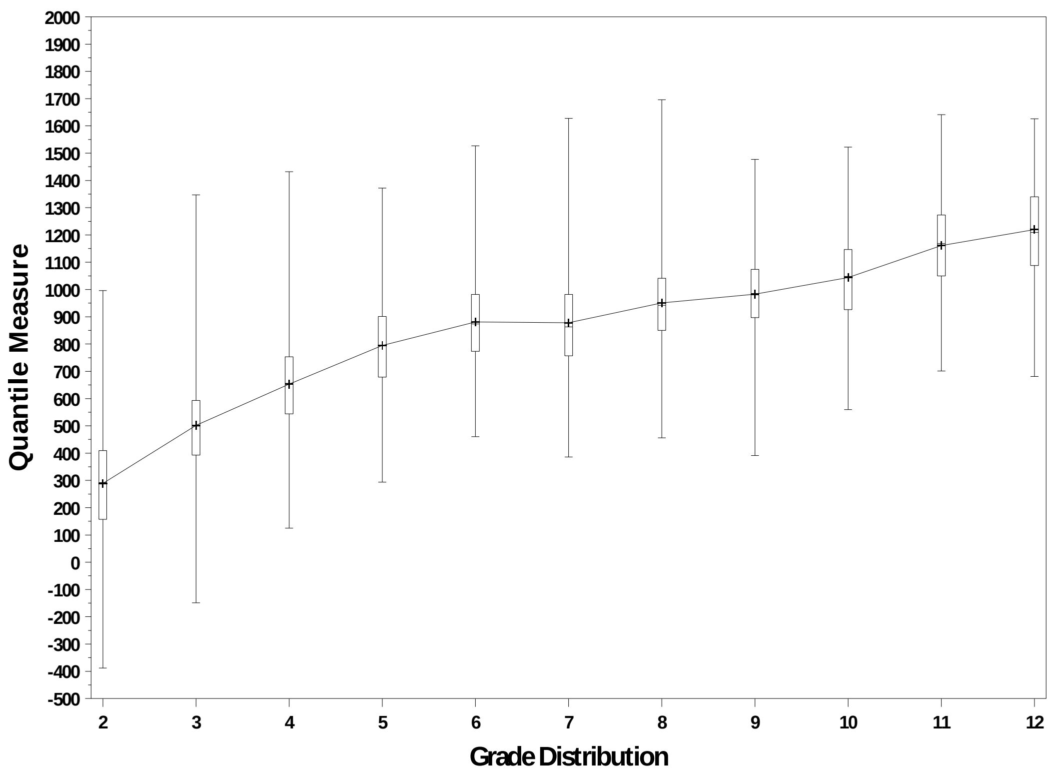
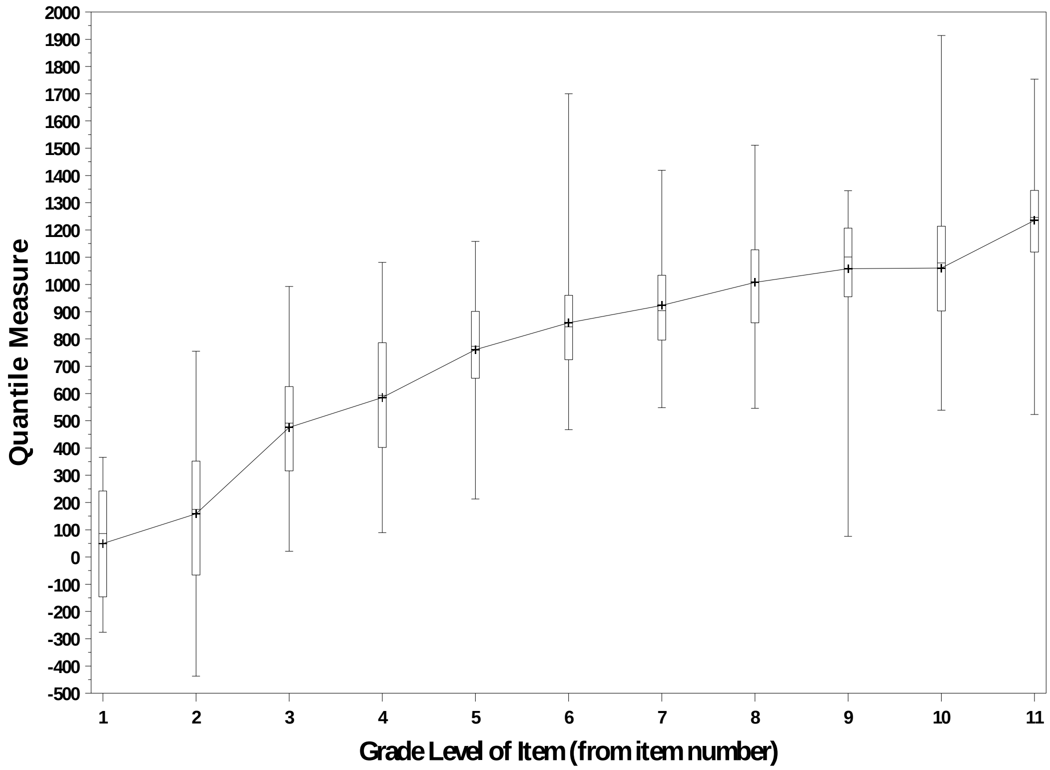


Figure 4 shows the distribution of item difficulties based on the final sample of students. For this analysis, missing data were treated as "skipped" items and not counted as wrong. There is a gradual increase in difficulty when items are sorted by level of test for which the items were written. This distribution appears to be non-linear, which is consistent with other studies. The correlation between the grade level for which the item was written and the Quantile measure of the item was 0.80.

Figure 4. Box-and-whisker plots of the Rasch difficulty estimates of the 685 Quantile Framework items for the final sample of students (N = 9,176).



The field testing of the items written for the Quantile framework indicates a strong correlation between the grade level of the item and the item difficulty.

The Specification of the Quantile Scale

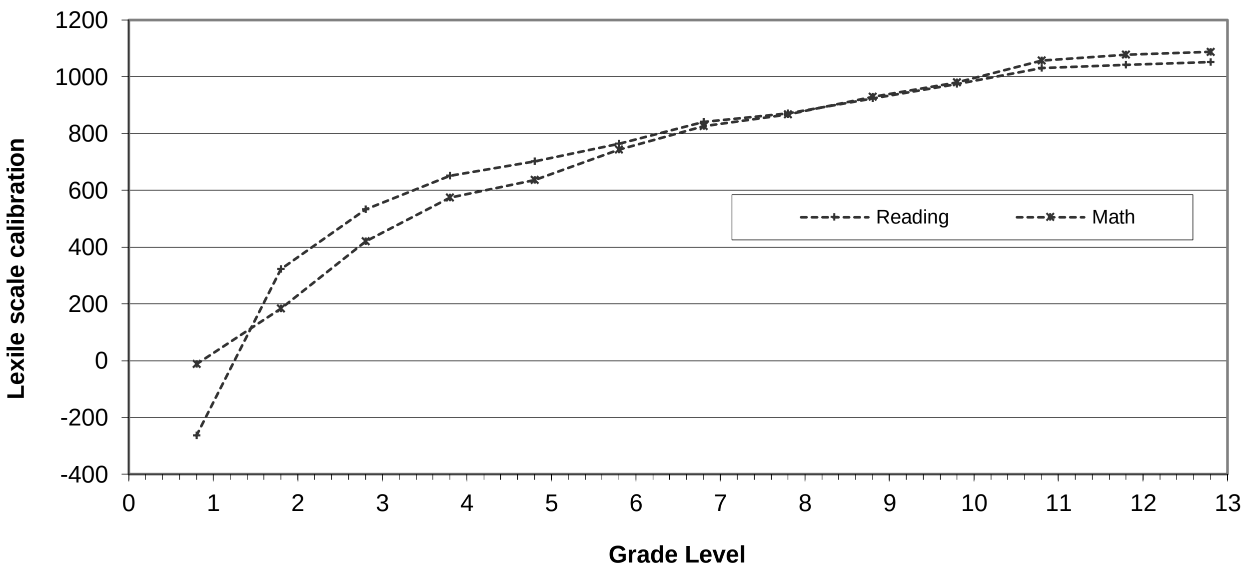
In developing the Quantile scale, two features of the scale were needed: (1) the scale multiplier (conversion factor), and (2) the anchor point.

As described in the previous section, the Rasch item response theory model (Wright and Stone, 1979) was used to estimate the difficulties of items and the abilities of persons on the logit scale. The calibrations of the items from the Rasch model are objective in the sense that the relative difficulties of the items will remain the same across different samples of persons (specific objectivity). When two items are administered to the same person it can be determined which item is harder and which one is easier. This ordering should hold when the same two items are administered to a second person. If two different items are administered to the second person, there is no way to know which set of items is harder and which set is easier.

The problem is that the location of the scale is not known. General objectivity requires that scores obtained from different test administrations be tied to a common zero—absolute location must be sample independent (Stenner, 1990). To achieve general objectivity, the theoretical logit difficulties must be transformed to a scale where the ambiguity regarding the location of zero is resolved.

The first step in developing the Quantile scale was to determine the conversion factor used to go from logits to Quantile measures. Based on prior research with reading and the Lexile scale, the decision was made to examine the relationship between reading and mathematics scales used with other assessments. The median scale score for each grade level on a norm-referenced assessment linked with the Lexile scale is plotted in *Figure 5* using the same conversion equation for both reading and mathematics.

Figure 5. Relationship between reading and mathematics scale scores on a norm-referenced assessment linked to the Lexile scale in reading.



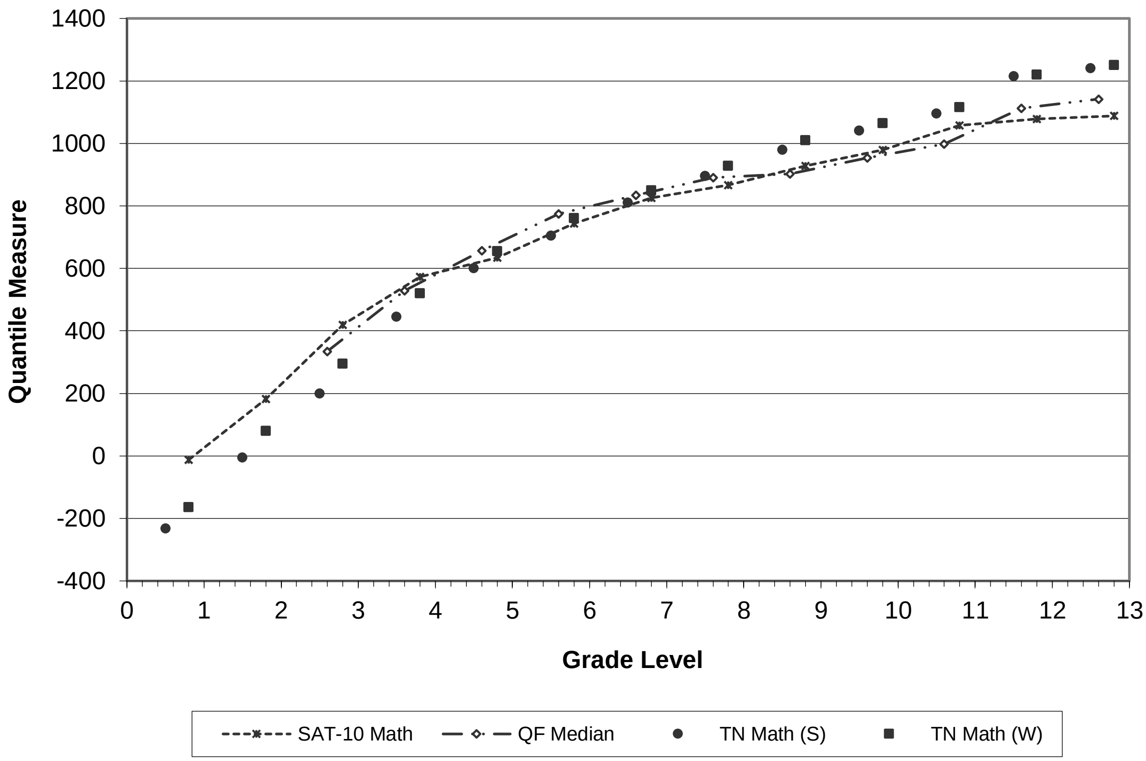
Based on an examination of *Figure 5*, it was concluded that the same conversion factor of 180 that is used with the Lexile scale could be used with the Quantile scale. Both sets of data exhibited a similar pattern across grades.

The second step in developing the Quantile scale with a fixed zero was to identify an anchor point for the scale. Given the number of students at each grade level in the field study, it was concluded that the scale should be anchored at grade 4 or 5 (middle of grade span typically tested by state assessment programs). Median performance at the end of grade 3 on the Lexile scale is 590L. The

Quantile Framework field study was conducted in February and this point would correspond to six months (0.6) through the school year. Median performance at the end of grade 4 on the Lexile scale is 700L. To determine the location of the scale, 66 Quantiles were added to the median performance at the end of grade 3 to reflect the growth of students in grade 4 prior to the field study (700 – 590 = 110; 110 × 0.6 = 66).

Therefore, the value of 656Q was used for the location of grade 4 median performance. The anchor point was validated with other assessment data and collateral data from the Quantile Framework field study (see *Figure 6*).

Figure 6. *Relationship between grade level and mathematics performance on the Quantile Framework field study and other mathematics assessments.*



As a result of the above analyses, a linear equation of the form

$$[(\text{Logit} - \text{Anchor Logit}) \times 180] + 656 = \text{Quantile measure} \qquad \text{(Equation 5)}$$

was developed to convert logit difficulties to Quantile calibrations where the anchor logit is the median for Grade 4 in the Quantile Framework Field Study.

Quantile Skill and Concept (QSC) Measures

The next step was to use the Quantile Framework to estimate the Quantile measure of each QSC. Having a measure for each QSC on the Quantile scale will then allow the difficulty of skills and concepts and the complexity of other resources to be evaluated. The Quantile measure of a QSC estimates the solvability, or a prediction of how difficult the skill or concept will be for a learner.

The QSCs also fall into Knowledge Clusters along a content continuum. Recall that the Quantile Framework is a content taxonomy of mathematical skills and topics. Knowledge Clusters are a family of skills, like building blocks, that depend one upon the other to connect and demonstrate how comprehension of a mathematical topic is founded, supported, and extended along the continuum. The Knowledge Clusters illustrate the interconnectivity of the Quantile Framework and the natural progression of mathematical skills (content trajectory) needed to solve increasingly complex problems (Hudnutt, 2012).

The Quantile measures and Knowledge Clusters for QSCs were determined by a group of three to five subject-matter experts (SMEs). Each SME has had classroom experience at multiple developmental levels, has completed graduate-level courses in mathematics education, and understands basic psychometric concepts and assessment issues.

For the development of Knowledge Clusters, certain terminology was developed to describe relationships between the QSCs.

- A **focus QSC** is the skills and concept that is the focus of instruction.
- A **prerequisite QSC** is a QSC that describes a skill or concept that provides a building block necessary for another QSC. For example, adding single-digit numbers is a prerequisite for adding two-digit numbers.
- A **supporting QSC** is a QSC that describes associated skills or knowledge that assists and enriches the understanding of another QSC. For example, two supporting QSC are multiplying two fractions and determining the probability of compound events.
- An **impending QSC** describes a skill or concept that will further augment understanding, building on another QSC. An impending QSC for using division facts is simplifying equivalent fractions.

Each focus QSC was classified with prerequisite QSCs and supporting QSCs or was identified as a foundational QSC. As a part of a taxonomy, QSCs are either a single link in a chain of skills that lead to the understanding of larger mathematical concepts, or they are the first step toward such an understanding. A QSC that is classified as foundational requires only general readiness to learn.

The SMEs examined each QSC to determine where the specific QSC comes in the content continuum based on their classroom experience, instructional resources (e.g., textbooks), and other curricular frameworks (e.g., NCTM Standards). The process called for each SME to independently review the QSC and develop a draft Knowledge Cluster. The second step consisted of the 3-5 SMEs meeting and reviewing the draft clusters. Through discussion and consensus, the SMEs developed the final Knowledge Cluster.

Once the Knowledge Cluster for a QSC was established, the information was used when determining the Quantile measure of a QSC, as described below. If necessary, Knowledge Clusters are reviewed and refined if the Quantile measures of the QSs in the cluster are not monotonically increasing (steadily increasing) or there is not an instructional explanation for the pattern.

The Quantile Framework is a theory-referenced measurement system of mathematical understanding. As such, a QSC Quantile measure represents the "typical" difficulty of all items that could be written to represent the QSC and the collection of items can be thought of as an *ensemble* of the all of the items that could be developed for a specific skill or concept. During 2002, Stenner, Burdick, Sanford, and Burdick (2006) conducted a study to explore the "ensemble" concept to explain differences across reading items with The Lexile Framework for Reading. The theoretical Lexile measure of a piece of text is the mean theoretical difficulty of all items associated with the text. Stenner and his colleagues state that the "Lexile Theory replaces statements about individual items with statements about ensembles. The ensemble interpretation enables the elimination of irrelevant details. The extra-theoretical details are taken into account jointly, not individually, and, via averaging, are removed from the data text explained by the theory" (p. 314). The result is that when making text-dependent generalizations, text readability can be measured with high accuracy and the uncertainty in expected comprehension is largely due to the unreliability in reader measures.

To determine the Quantile measure of a QSC, actual performance by examinees is used. While expert judgment alone could be used to scale the QSCs, empirical scaling is more replicable. Items and resulting data from two national field studies were used in the process:

- Quantile Framework field study (685 items, N = 9,647, grades 2 through Algebra II) which is described earlier in this section; and
- *PASeries* Mathematics field study (7,080 items, N = 27,329, grades 2 through 9/Algebra I) which is described in the *PASeries* Mathematics Technical Manual (MetaMetrics, 2005).

The items initially associated with each QSC were reviewed by SMEs and accepted for inclusion in the set of items, moved to another QSC, or not included in the set. The following criteria were used:

- Psychometric (responded to by at least 50 examinees, administered at the target grade level, point-biserial correlation greater than or equal to 0.16);
- Matched grade level of introduction of concept/skill from national review of curricular frameworks (described on pages 3 and 4); and
- Appropriate for instruction of concept (first nights homework; from the A and B sections of the lesson problems) based on consensus of the SMEs.

Once the set of items meeting the inclusion criteria is identified, the set of items is reviewed to ensure that the curricular breadth of the QSC is covered. If the group of SMEs considers the set of items to be acceptable, then the Quantile measure of the QSC is calculated. The Quantile measure of a QSC is defined as the mean Quantile measure of items that met the criteria.

The final step in the process is to review the Quantile measure of the QSC in relationship to the Quantile measures of the QSCs identified as precursory and supporting to the QSC. If the group of SMEs does not consider the set of items to be acceptable, then the Quantile measure of the QSC is estimated and assigned a Quantile zone. (Quantile zone is the suggested range of Quantiles at which the student is ready for instruction. The Quantile Range for a student is from 50Q above her or his Quantile measure to 50Q below.) By assigning a Quantile zone instead of a Quantile measure to these QSCs, the SMEs are able to provide a valid estimate of the skill or concept's difficulty.

In 2007, with the extension of the Quantile Framework to include Kindergarten and Precalculus, the Quantile measures of the QSCs were reviewed. Where additional items had been tested and the data was available, estimated QSC Quantile measures were calculated. In 2014, a large data set was analyzed to examine the relationship between the original QSC Quantile measures and empirical QSC means from the items administered. The overall correlation between QSC Quantile measures and empirically estimated Quantile measures was 0.98 (N = 7,993 students). Based on the analyses, 12 QSCs were identified with larger-than-expected deviations given the "ensemble" interpretation of a QSC Quantile measure. Each QSC was reviewed in terms of the items that generated the data, linking studies where the QSC was employed, and data from other assessments developed employing the Quantile Framework. Of the 12 QSCs identified, it was concluded that the Quantile measure of nine of the QSCs should be recalculated. Five of the QSCs are targeted for Kindergarten and Grade 1 and the current data set provided data to calculate a Quantile measure (the Quantile measure for the QSC had been previously estimated). The other four QSC Quantile measures were revised because the type of "typical" item and the technology used to assess the skill or concept had shifted from the time that the QSC Quantile measure was established in 2004 (QSCs: 79, 654, 180, and 217). Three of the QSC Quantile measures were not changed (QSC: 134, 604, 408) because (1) some of the items did not reflect the intent of the QSC, or (2) not enough items were tested to indicate that the Quantile measure should be recalculated.

In 2019, the Quantile Framework taxonomy was extended to include advanced statistics and calculus. A total of 74 QSCs were developed (advanced statistics, 29 QSCs; and calculus, 45 QSCs). Five to six items were developed for each new QSC to span the range of content and cognitive complexity and then field tested. A total of 1,170 students enrolled in advanced mathematics or AP calculus or statistics classes participated in the field study. All items were analyzed for psychometric quality and calibrated to the Quantile scale. QSC measures were estimated based on the mean and standard deviation of the item difficulties. QSCs with large item difficulty standard deviations or insufficient number of items with adequate psychometric quality (e.g., p-values below .10 or above .95, or point-measure correlation below 0.16) were not estimated (QSCs: 2033, 2037, and 2067). Based on the analysis, a total of 71 QSCs were added to the Quantile Framework and the QSC Quantile measures ranged from 1070Q to 1670Q.

Validity of the Quantile Framework for Mathematics

Validity is the extent to which a test measures what its authors or users claim it measures. Specifically, test validity concerns the appropriateness of inferences "that can be made on the basis of observations or test results" (Salvia and Ysseldyke, 1998, p. 166). The 2014 *Standards for Educational and Psychological Testing* (American Educational Research Association, American Psychological Association, and National Council on Measurement in Education) state that "validity refers to the degree to which evidence and theory support the interpretations of test scores for proposed uses of tests" (p. 11). In other words, a valid test measures what it is supposed to measure.

Stenner, Smith, and Burdick state that "[t]he process of ascribing meaning to scores produced by a measurement procedure is generally recognized as the most important task in developing an educational or psychological measure, be it an achievement test, interest inventory, or personality scale" (1983). For the Quantile Framework, which measures student understanding of mathematical skills and concepts, the most important aspect of validity that should be examined is construct-identification validity. This global form of validity encompassing content-description and criterion-prediction validity may be evaluated for The Quantile Framework for Mathematics by examining how well Quantile measures relate to other measures of mathematical achievement.

Relationship of Quantile Measures to Other Measures of Mathematical Understanding

Scores from tests purporting to measure the same construct, for example "mathematical achievement," should be moderately correlated (Anastasi, 1982). The Quantile Framework for Mathematics has been linked with numerous standardized tests of mathematics achievement. When assessment scales are linked, a common frame of reference can be used to interpret the test results. This frame of reference can be "used to convey additional normative information, test-content information, and information that is jointly normative and content-based. For many test uses … [this frame of reference] conveys information that is more crucial than the information conveyed by the primary score scale" (Petersen, Kolen, and Hoover, 1993, p. 222).

Table 6 presents the results from linking studies conducted with the Quantile Framework. For each of the tests listed, student mathematics scores were reported using the test's scale, as well as by Quantile measures. This dual reporting provides a rich, criterion-related frame of reference for interpreting the standardized test scores. Each student who takes one of the standardized tests can receive, in addition to norm- or criterion-referenced test results, information related to the specific QSCs on which he or she is ready to be instructed. *Table 6* also shows that measures derived from the Quantile Framework are more than moderately correlated to other measures of mathematical understanding.

Table 6. Results from linking studies conducted with the Quantile Framework.

Standardized Test	**Grades in Study**	***N***	**Correlation Between Test Score and Quantile measure**
Mississippi Curriculum Test, Mathematics (MCT)	2 – 8	7,039	0.89
TerraNova (CTB/McGraw-Hill)	3, 5, 7, 9	6,356	0.92
Texas Assessment of Knowledge and Skills (TAKS)	3 – 11	14,286	0.69 to 0.78*
Proficiency Assessments for Wyoming Students (PAWS)	3, 5, 8, and 11	3,923	0.87
Progress Towards Standards (PTS3)	3-8 and 10	8,544	0.86 to 0.90*
Progress in Maths (PiM – GL Assessments)	1 – 8	3,183	0.71 to 0.81*
North Carolina End-of-Grade/End-of-Course Tests (NC EOG/NC EOC)	3, 5, 7, A1, G, and A2	5,069	0.88 to 0.90*
Comprehensive Testing Progressing (CPT 4 – ERB)	3, 5, and 7	953	0.87 to 0.90
Kentucky Core Content Tests (KCCT)	3 - 8 and 11	12,660	0.80 to 0.83*
Oklahoma Core Competency Tests (OCCT)	3 – 8	5,649	0.81 to 0.85*
Iowa Assessments	2, 4, 6, 8, and 10	7,365	0.92
Virginia Standards of Learning (SOL)	3-8, A1, G, and A2	12,470	0.86 to 0.89*
Kentucky Performance Rating for Educational Progress (K-PREP)	3 – 8	6,859	0.81 to 0.85*
North Carolina ACT	11	3,320	0.90
North Carolina READY End-of-Grade/End-of-Course Tests (NC EOG/NC EOC)	3, 4, 6, 8, and A1/I1	10,903	0.87 to 0.90*
aimsweb – Math Concepts and Applications (Pearson)	2 – 8	3,262	0.87

Notes: * TAKS, PTS3, PiM, NCEOC, KCCT, OCCT, K-PREP, SOL, and NC READY were not vertically scaled; separate linking equations were derived for each grade/course.

Multidimensionality of Quantile Framework Items

Test dimensionality is defined as the minimum number of abilities or constructs measured by a set of test items. A construct is a theoretical representation of an underlying trait, concept, attribute, process, and/or structure that a test purports to measure (Messick, 1993). A test can be considered to measure one latent trait, construct, or ability (in which case it is called unidimensional); or a combination of abilities (in which case it is referred to as multidimensional). The dimensional structure of a test is intricately tied to the purpose and definition of the construct to be measured. It is also an important factor in many of the model(s) used in data analyses. Though many of the models assume unidimensionality, this assumption cannot be strictly met because there are always other cognitive, personality, and test-taking factors that have some level of impact on test performance (Hambleton and Swaminathan, 1985).

The complex nature of mathematics and the curriculum standards most states have adopted also contribute to unintended dimensionality. Application and process skills, the reading demand of items, and the use of calculators could possibly add features to an assessment beyond what the developers intended. In addition, the *NCTM Standards*, upon which many states have based curricula, describe the growth of students' mathematical development across five content standards: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. These standards, or sub-domains of mathematics, are useful in organizing mathematics instruction in the classroom. These standards could represent different constructs and thereby introduce more sources of dimensionality to tests designed to assess these standards.

Study 1 – Comparison of Mathematics with Reading. The multidimensionality of the Quantile scale was examined using the Principal Components Analysis of Residuals in Winsteps (PRCOMP=S) (MetaMetrics, 2014). A three-step process was undertaken in order to examine the results and provide a context for interpreting the results.

The first step in the process was to run the Principal Components Analysis on all Quantile Framework field study items (N = 898). Next, the residual matrix was factor analyzed. The variance that is unexplained by the first factor (the Rasch measurement model) is 0.2% of the residual variance or 2.5 items of information. Based upon this set of data, it cannot be concluded that mathematics achievement as measured by the Quantile scale is multidimensional. The results supported the use of a unidimensional item response model on the items.

Next, the items were ordered by factor loading. Based on an examination of the item names with strand listed first, there did not appear to be any effect of strand and the items measured a general construct of mathematics. The results showed that items from all strands loaded most highly on the first (general factor) and no set of items from a particular strand loaded on a specific factor. As a sub-analysis, items from the Geometry and Algebra and Algebraic Thinking strands were analyzed. It was hypothesized, that if multi-dimensionality were to be evidenced in the data, this would be the most likely contrast. The Rasch model explained 54.1% of the variance in the Geometry and Algebra and Algebraic Thinking items. The results from the study are consistent with the interpretation of a single construct for each of the analyses (mathematics).

The third step was to examine the results of reading (considered a unidimensional construct) with the mathematics results. The Rasch model explained 60.6% of the variance in the reading comprehension items. Along with the results presented in the first two steps of the process, these data are consistent with the use of a unidimensional item response theory model for each of the analyses (reading and mathematics).

Study 2 – Burg (2007). A study conducted by Burg (2007) analyzed the dimensional structure of mathematical achievement tests aligned to the NCTM content standards. Since there is not a consensus within the measurement community on a single method to determine dimensionality, Burg employed four different methods for assessing dimensionality:

- exploring the conditional covariances (DETECT)
- assessment of essential unidimensionality (DIMTEST)
- item factor analysis (NOHARM) and
- principal component analysis (WINSTEPS)

All four approaches have been shown to be effective indices of dimensional structure. Burg analyzed Grades 3 through 8 data from the Quantile Framework Field Study previously described.

Each set of on-grade items for a test form from Grades 3 through 8 were analyzed for possible sources of dimensionality related to the five mathematical content strands. The analyses were also used to compare test structures across grades. The results indicated that although mathematical achievement tests for Grades 3 through 8 are complex and exhibit some multidimensionality, the sources of dimensionality are not related to the content strands. The complexity of the data structure, along with the known overlap of mathematical skills, suggests that mathematical achievement tests could represent a fundamentally unidimensional construct. While these sub-domains of mathematics are useful for organizing instruction, developing curricular materials such as textbooks, and describing the organization of items on assessments, they do not describe a significant psychometric property of the test or impact the interpretation of the test results. Mathematics, as measured by the Quantile Framework, can be described as one construct with various sub-domains.

These findings support the NCTM Connections Standard, which states that all students (prekindergarten through Grade 12) should be able to make and use connections among mathematical ideas and see how the mathematical ideas interconnect. Mathematics can be best described as an interconnection of overlapping skills with a high degree of correlation across the mathematical topics, skills, and strands.

Furthermore, these finding support the goals of the Common Core State Standards for Mathematics by providing the foundations of a growth model by which a single measure can inform progress toward college and career readiness.

Study 3 – Hennings and Simpson (2012). Results from Hennings and Simpson (2012) also suggest that the mathematics assessments used in MetaMetrics' linking studies are functionally unidimensional. Data from a Quantile Framework linking study involving the end-of-grade tests from a Southeastern state was examined. Scored student responses to items on the combined Quantile Linking Test and the state end-of-grade test were used. The end-of-grade tests had three polytomous items worth two points each on the forms for Grades 3 through 8, and one polytomous item worth four points on the forms for Grades 4 through 8. The remaining items on both tests were dichotomous and scored 0/1. *Table 7* shows the number of students and the number of items, combined and by test, for each grade.

Table 7. Number of items included in analyses.

Grade	*N* of Students	Quantile Linking Test	End-of-Grade Test	Total
3	897	40	47	87
4	1,161	42	48	90
5	1,029	46	48	94
6	1,327	44	48	92
7	1,475	43	48	91
8	933	47	48	95

The polychoric item correlation matrix was analyzed for each test and grade. Because the principal components method of factor extraction in SAS does not require a positive-definite correlation matrix as input, principal component analyses were conducted instead of factor analyses.

The results support treating the data as unidimensional. The first component was dominant in all analyses. The first eigenvalue accounted for greater than 20% of the total variance in the analyses. Ratios of first-to-second eigenvalues ranged from approximately 6 to slightly over 9 (Gorsuch, 1983; Reckase, 1979). Secondary dimensions, i.e., the second and third components, accounted for approximately 5 - 6.5% of the total variance for each grade. *Table 8* lists the eignevalues for the first five principal components by grade, *Table 9* shows the ratios of first-to-second eigenvalues, and *Table 10* shows the proportion of variance accounted for by the first five principal components for each grade.

Table 8. Eigenvalues for the first five principal components, by grade.

	Principal Components				
Grade	**1**	**2**	**3**	**4**	**5**
3	24.152	3.463	2.411	2.253	2.011
4	23.252	3.637	2.257	1.894	1.829
5	22.770	3.222	2.407	2.239	1.935
6	21.400	3.058	2.297	2.185	1.866
7	23.919	3.922	2.442	1.744	1.648
8	24.572	2.654	2.152	2.076	1.914

Table 9. Ratio of the first-to-second eigenvalues, by grade.

Grade	Ratio
3	6.975
4	6.394
5	7.066
6	6.997
7	6.099
8	9.257

Table 10. Proportion of variance explained for the first five principal components, by grade.

	Principal Components				
Grade	**1**	**2**	**3**	**4**	**5**
3	0.278	0.040	0.028	0.026	0.023
4	0.258	0.040	0.025	0.021	0.020
5	0.242	0.034	0.026	0.024	0.021
6	0.233	0.033	0.025	0.024	0.020
7	0.263	0.043	0.027	0.019	0.018
8	0.259	0.028	0.023	0.022	0.020

Across the three studies, the results are consistent with the measurement of unidimensional constructs. The proportion of variance accounted for by the first factor in the principal components analyses consistently identifies a dominant factor. Additionally, when Burg (2007) employed multiple evaluative methods, the results suggested that content strands do not account for additional variance. Although content specifications are an important aspect of domain coverage and test construction, the evidence suggests that there is still a larger general mathematical factor that dominates student responses. Additionally, findings suggest that assessments are constructed in a fashion that supports a unidimensional construct with respect to its measurement properties (Hennings & Simpson, 2012). In conclusion, the studies suggest that while the mathematical content developed represents a complex set of concepts and skills, statistically speaking the construct is consistently represented by a unidimensional scale.

College and Career Preparedness in Mathematics

There is increasing recognition of the importance of bridging the gap that exists between K-12 and higher education and other postsecondary endeavors. Many state and policy leaders have formed task forces and policy committees such as P-20 councils.

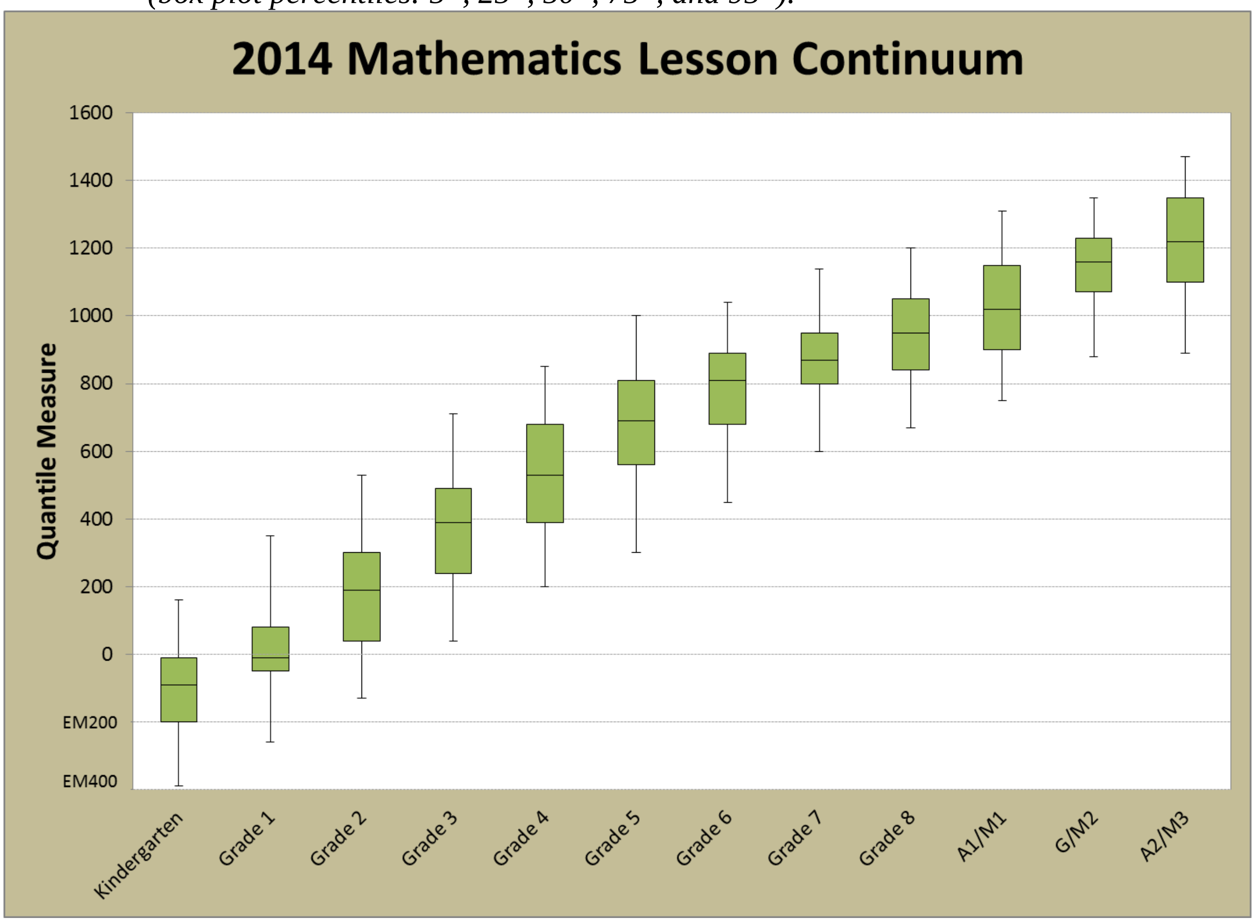
The Common Core State Standards for Mathematics (CCSSM) were designed to enable all students to become college and career ready by the end of high school while acknowledging that students are on many different pathways to this goal: "One of the hallmarks of the Common Core State Standards for Mathematics is the specification of content that all students must study in order to be college and career ready. This 'college and career ready line' is a minimum for all students" (NGA Center & CCSSO, 2010a, p. 4). The CCSS for Mathematics suggest that "college and career ready" means completing a sequence that covers Algebra I, Geometry, and Algebra II (or equivalently, Integrated Mathematics 1, 2, and 3) during the middle school and high school years; and, leads to a student's promotion into more advanced mathematics by their senior year. This has led some policy makers to generally equate the successful completion of Algebra II as a working definition of college and career ready. Exactly how and when this content must be covered is left to the states to designate in their implementations of the CCSS for Mathematics throughout K-12.

The *mathematical demand* of a mathematical textbook (in Quantile measures) quantitatively defines the level of mathematical achievement that a student needs in order to be ready for instruction on the mathematical content of the textbook. Assigning QSC(s) and Quantile measures to a textbook is done through a calibration process. Textbooks were analyzed at the lesson level and the calibrations were completed by subject matter experts (SMEs) experienced with the

Quantile Framework and with the mathematics taught in mathematics classrooms. The intent of the calibration process is to determine the mathematical demand presented in the materials. Textbooks contain a variety of activities and lessons. In addition, some textbook lessons may include a variety of skills. Only one Quantile measure is calculated per lesson and is obtained through analyzing the Quantile measures of the QSCs that have been mapped to the lesson. This Quantile measure represents the composite task demand of the lesson.

MetaMetrics has calibrated more than 41,000 instructional materials (e.g., textbook lessons, instructional resources) across the K-12 mathematics curriculum (Smith & Turner, 2012). *Figure 7* shows the continuum of calibrated textbook lessons from Kindergarten through Algebra II/Math 3 from 27,630 lessons (370 test books) from materials published since 2005 (Sanford-Moore, Williamson, Bickel, Koons, Baker, and Price, 2014).

Figure 7. A continuum of mathematical demand for Kindergarten through Precalculus textbooks (box plot percentiles: 5th, 25th, 50th, 75th, and 95th).



In 2016, Williamson, Sanford-Moore, and Bickel began the examination of the mathematics demands of college and careers to answer the question, “What mathematics must a student be capable of performing to be ready for college or career?” To address this question, the mathematical concepts and skills that students are likely to encounter as they begin their postsecondary education and/or enter the workplace were examined. For college readiness, being ready for instruction in the types of courses typical of those beyond high school graduation requirements and of first year college were examined (e.g., precalculus, trigonometry). For career readiness, competently performing the mathematics content required for a high school diploma (e.g., Algebra I content, Algebra II content) was examined. In this research, "completely perform" was defined as 75% understanding of the mathematics skills and concepts. The range (interquartile range) of student mathematical ability associated with being ready for the mathematics demands of college and careers is 1220Q to 1440Q, with a median of 1350Q.

Description of the LevelSet Math Assessments

Achieve3000 Math consists of three LevelSet Math assessments for each grade level. The first LevelSet Math assessment is called a Beginning of Year (BOY) form and the other two LevelSet Math assessments are called Middle of Year (MOY) and End of Year (EOY). Typically the Beginning of Year form is administered at the start of school (August 1 to September 30), the Middle of Year form is administered at least 9 weeks later (December 1 to January 31) and the End of Year form is administered at the end of the school year (May 1 to June 30). The tests are untimed, but each is designed to take between 30 and 35 minutes for a student to complete. The results can be used to determine a student's readiness for mathematics instruction on grade-level skills and concepts and place the student at the appropriate mathematics ability level. All three test forms at each grade level have 30 multiple-choice items.

The LevelSet Math assessments can be used by educators for screening and placement and to inform and plan targeted instruction. Through the use of the Quantile Framework, Achieve3000, Inc. provides fast and effective tools for:

- instructional placement,
- flexible group decisions, and
- connecting assessment and instruction for all students.

Student results are reported as a Quantile measure. There are many reasons to use scale scores, in this case Quantile measures, rather than raw scores to report test results. Scale scores overcome the disadvantage of many other types of scores (e.g., percentiles and raw scores), in that equal differences between scale score points represent equal differences in ability. Each item on a test has a unique level of difficulty; therefore, answering 17 items correctly on one form of a test may require a slightly different level of ability than answering 17 items correctly on another form of the test. In contrast, receiving a scale score (Quantile measure) of 775 on one form of a test represents a similar level of mathematical ability as receiving a scale score (Quantile measure) of 775 on another form of the test.

The typical range of the Quantile Scale is from below 0Q to above 1600Q. There is a not an explicit bottom or top to the scale, but rather an anchor point on the scale that describes a specific level of mathematical understanding. The Quantile Map, a graphic representation of the Quantile Scale from EM100Q to 1500Q+, provides a context for examining and describing mathematical understanding (see Appendix A). Quantile student measures are reported in 5-unit intervals. Scores at or below 0Q are reported as EMxxxQ (where "EM" represents Emerging Mathematician).

Periodic assessments with vertical scales provide unique interpretability with respect to student growth. Furthermore, conjoint measurement yields instructionally useful information for students, parents and teachers particularly in support of differentiation and targeted learning experiences. Viewing student growth in the context of postsecondary mathematics requirements further empowers students to plan for their future. The Quantile Framework theory of action posits that such feedback contributes to student motivation to aspire toward their postsecondary endeavors in a more thoughtful and systematic way (MetaMetrics, 2016).

LevelSet Math Assessment Design

The LevelSet Math assessments are designed so that students may take a test at the beginning of the year (BOY), middle of the year (MOY), and end of the year (EOY). Each student who takes an assessment will receive a Quantile measure.

The LevelSet Math assessments focus specifically on Kindergarten through Grades 11/12. For each grade, Beginning of Year forms were created covering prior grade skills and concepts. The purpose of the Beginning of Year form is to determine a general Quantile level for the student to set a baseline for the growth in Quantile measures that will occur throughout the year. The second and third test forms were designed to monitor progress during the remainder of the school year and are constructed to be administered at two different times throughout the year: middle-of-the-year and end-of-the-year. Middle of Year forms cover prior grade and on grade level skills and concepts, and End of Year forms cover on grade level skills and concepts. LevelSet Math assessments for each grade consist of 30 multiple-choice items from the Quantile Item Bank and cover the QSCs associated with the Common Core State Standards for Mathematics (CCSS) curriculum standards.

On each form, if a student has answered five items in less than two minutes and provided at least three incorrect answers, the LevelSet Math assessment will stop and notify the teacher of the respective student. The teacher will be notified and asked to either:

1. readminister the LevelSet Math assessment restarting the student from item one, or
2. continue to item six.

The teacher must choose one of the two options in order for the student to continue completing the LevelSet Math assessment.

The Kindergarten through Grade 4 LevelSet Math assessments do not require students to have access to calculators. In Grade 5, the Beginning of Year LevelSet Math assessment does not require students to have access to calculators, while the Middle of Year and End of Year LevelSet Math assessments allow students to have access to calculators on designated items. In Grades 6 through 8, all of the LevelSet Math assessments allow students to have access to calculators on designated items. In Grades 9 through 11/12, the LevelSet Math assessments allow students to have access to calculators on all items.

Some, but not all, of the items within the LevelSet Math assessments allow students to have access to a formula sheet. Students will have access to a grade-appropriate formula sheet for items that require one.

Interpreting LevelSet Math Assessment Results

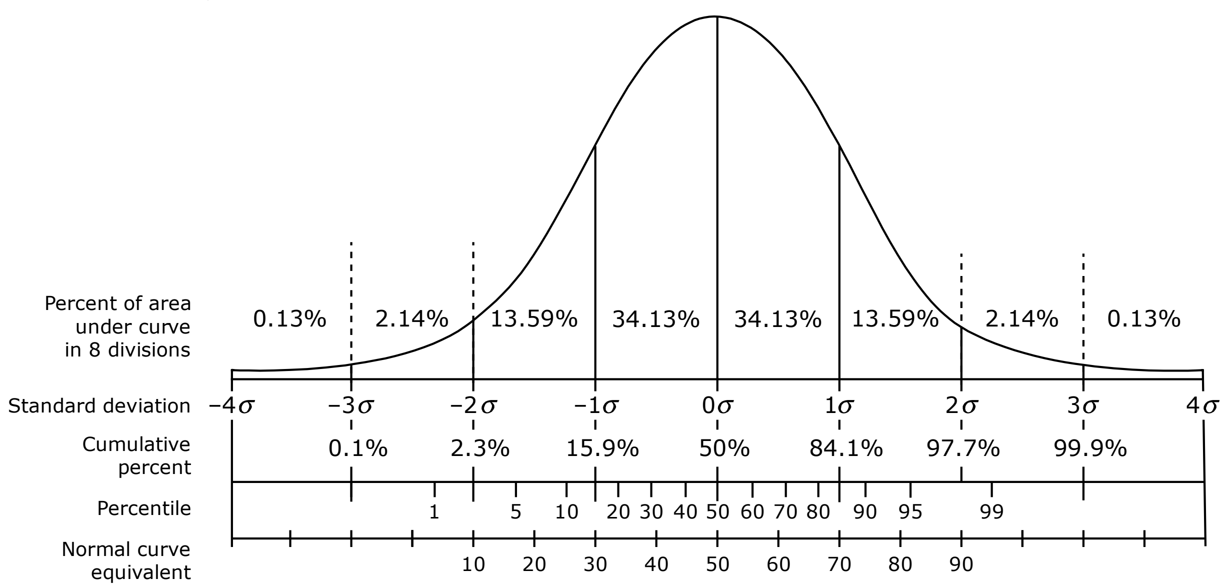
Results from the LevelSet Math assessments are reported as scale scores (Quantile measures). This scale extends from Emerging Mathematician (below 0Q) to above 1600Q. The score is determined by the difficulty of the items a student answered both correctly and incorrectly. Scale scores can be used to report the results of both criterion-referenced tests and norm-referenced tests. The student's Quantile measure from LevelSet Math assessments are used to determine the student's readiness for mathematics instruction and to place the student at an appropriate starting point in the Achieve3000 Math's instructional program.

Achieve3000 Math provides both criterion-referenced and norm-referenced interpretations of the Quantile measures. Criterion-referenced interpretations of test results provide a rich frame of reference that can be used to guide instruction for optimal student mathematics growth. While norm-referenced interpretations of test results are often required for accountability purposes, they indicate only how well the student is achieving in mathematics in relation to how other, similar students are achieving.

Norm-Referenced Interpretations using Percentiles

A norm-referenced interpretation of a test score expresses how a student performed on the test compared to other students of the same age or grade. Norm-referenced interpretations of reading test results, however, do not provide any information about what a student can or cannot read. For accountability purposes, percentiles, normal curve equivalents (NCEs), and stanines are used to report test results when making comparisons (norm-referenced interpretations). For a comparison of these measures, refer to *Figure 8*.

Figure 8. Normal distribution of scores described in scale scores, percentiles, stanines, and NCEs.



The percentile rank of a score indicates the percentage of scores less than or equal to that score. Percentile ranks range from 1 to 99. For example, if a student scores at the 65th percentile, it means that he or she performed as well as or better than 65% of the norm group. Real differences in performance are greater at the ends of the percentile range than in the middle. Percentile ranks of scores can be compared across two or more distributions; percentile ranks cannot be used to determine differences in relative rank due to the fact that the intervals between adjacent percentile ranks do not necessarily represent equal raw score intervals. Note that the percentile rank does not refer to the percentage of items answered correctly.

The normative information for The Quantile Framework for Mathematics is based on linking studies conducted with the Quantile Framework and the results of assessments that report directly in the Quantile metric (*N* = 3,213,563). The sample included students in Kindergarten through Grade 12 from 38 states, districts, or territories and who were tested from 2010 to 2016 (Grades 1-12) and 2016 to 2019 (Kindergarten). Of the students with gender information (29%), 51.0% of the students were male and 49.0% of the students were female. Of the students with race or ethnicity information (28%), the majority of the students in the norming sample were White (65.7%), with 5.1% African-American, 2.3% American Indian/Alaskan Native, 15.3% Hispanic, 5.8% Asian, and 5.5% Other. Of the students with data, 7.4 percent of the students were classified as "Needing Special Education Services." The 2020 Quantile norms have been validated in relation to a longitudinal sample of students across Grades 3 through 11 (*N* = 101,650). *Tables 11* through *13* present normative data at selected percentiles from the fall (August to November), winter (December to February), and spring (March through June) Quantile norms administration periods.

Table 11. Selected percentiles from the 2020 Quantile norms, fall test administration period.

Grade	**5th**	**10th**	**25th**	**50th**	**75th**	**90th**	**95th**
K	EM590Q	EM485Q	EM345Q	EM205Q	EM60Q	90Q	180Q
1	EM255Q	EM260Q	EM125Q	10Q	150Q	290Q	380Q
2	EM135Q	EM45Q	80Q	210Q	340Q	475Q	560Q
3	50Q	140Q	260Q	385Q	510Q	640Q	725Q
4	210Q	295Q	420Q	540Q	665Q	790Q	870Q
5	340Q	430Q	550Q	675Q	795Q	925Q	1005Q
6	440Q	525Q	645Q	765Q	890Q	1015Q	1095Q
7	535Q	620Q	740Q	865Q	985Q	1110Q	1190Q
8	615Q	700Q	825Q	945Q	1070Q	1195Q	1275Q
9	690Q	775Q	895Q	1020Q	1140Q	1270Q	1350Q
10	750Q	835Q	960Q	1080Q	1205Q	1335Q	1415Q
11/12	810Q	895Q	1015Q	1140Q	1265Q	1390Q	1470Q

Table 12. Selected percentiles from the 2020 Quantile norms, winter test administration period.

Grade	**5th**	**10th**	**25th**	**50th**	**75th**	**90th**	**95th**
K	EM550Q	EM450Q	EM305Q	EM160Q	EM10Q	140Q	235Q
1	EM295Q	EM200Q	EM70Q	65Q	200Q	340Q	430Q
2	EM85Q	5Q	130Q	260Q	390Q	520Q	605Q
3	95Q	185Q	305Q	430Q	555Q	680Q	765Q
4	250Q	335Q	455Q	575Q	700Q	825Q	910Q
5	365Q	450Q	570Q	695Q	815Q	940Q	1025Q
6	465Q	550Q	670Q	795Q	915Q	1040Q	1120Q
7	560Q	645Q	765Q	885Q	1010Q	1135Q	1215Q
8	635Q	720Q	845Q	965Q	1090Q	1215Q	1295Q
9	705Q	790Q	915Q	1035Q	1160Q	1285Q	1365Q
10	765Q	855Q	975Q	1100Q	1220Q	1350Q	1430Q
11/12	825Q	910Q	1030Q	1155Q	1280Q	1405Q	1485Q

Table 13. Selected percentiles from the 2020 Quantile norms, spring test administration period.

Grade	5th	10th	25th	50th	75th	90th	95th
K	EM515Q	EM410Q	EM265Q	EM115Q	35Q	190Q	290Q
1	EM235Q	EM140Q	EM10Q	120Q	255Q	390Q	475Q
2	EM30Q	55Q	180Q	310Q	435Q	565Q	650Q
3	140Q	225Q	350Q	470Q	595Q	725Q	805Q
4	285Q	370Q	490Q	615Q	740Q	865Q	945Q
5	390Q	475Q	595Q	715Q	835Q	960Q	1040Q
6	495Q	580Q	700Q	820Q	945Q	1070Q	1150Q
7	580Q	665Q	785Q	910Q	1030Q	1155Q	1240Q
8	655Q	740Q	865Q	985Q	1110Q	1235Q	1315Q
9	725Q	810Q	930Q	1055Q	1175Q	1305Q	1385Q
10	785Q	870Q	990Q	1115Q	1240Q	1365Q	1445Q
11/12	835Q	925Q	1045Q	1170Q	1295Q	1420Q	1500Q

A normal curve equivalent (NCE) is a normalized student score with a mean of 50 and a standard deviation of 21.06. NCEs range from 1 to 99. NCEs allow comparisons between different tests for the same student or group of students and between different students on the same test. NCEs have many of the same characteristics as percentile ranks, but have the additional advantage of being based on an interval scale. That is, the difference between two consecutive scores on the scale has the same meaning throughout the scale. NCEs are required by many categorical funding agencies (for example, Title I).

While not very useful at the student level, normative information can be useful (and often required) at the aggregate levels for program evaluation.

Criterion-Referenced Interpretations using Performance Standards

A growing trend in education is to differentiate between content standards—curricular frameworks that specify what should be taught at each grade level—and performance standards—what students must do to demonstrate proficiency with respect to the specific content. Increasingly, educators and parents want to know more than just how a student's performance compares with that of other students: they ask, "What level of performance does a score represent?" and "How good is good enough?"

Performance levels describe students' performance when instructed on grade-level appropriate skills and concepts. Combined with the student Quantile measure, the performance levels can be used to group students and determine appropriate instruction. Following the LevelSet Math assessments, students are classified into one of the following four student performance levels:

- **Level I (Falls Far Below)** -- Students performing at this level have *minimal or no* academic performance indicating an understanding and *little* display of the knowledge and skills included in state and national content standards. Students performing at this standard are *not on track* for likely success in the next grade or course.
- **Level II (Approaches)** -- Students performing at this level have *marginal/inconsistent* academic performance indicating a *partial* understanding and *limited* display of the knowledge and skills included in state and national content standards. Students

performing at this standard may be on track for likely success in the next grade or course with additional supports.

- **Level III (Meets)** -- Students performing at this level have satisfactory academic performance indicating a *solid* understanding and display of the knowledge and skills included in state and national content standards. Students performing at this standard are on track for likely success in the next grade or course.
- **Level IV (Exceeds)** -- Students performing at this level have *competent/superior* academic performance indicating an *advanced/in-depth* understanding and *exemplary* display of the knowledge and skills included in state and national content standards. Students performing at this standard are on track for success in the next grade or course.

The performance standards for LevelSet Math assessments were designed to be reflective of the performance standards set by states on state summative assessments and to describe whether the student would be on track for mathematics demands of the next grade or course. The following state assessment performance standards were examined to develop the Achieve3000 Math performance levels: California, Connecticut, Indiana, Minnesota, Missouri, Nevada, North Dakota, South Carolina, West Virginia, and Wyoming.

First, the bottom of the "Level II", "Level III", and "Level IV" performance levels for each grade were calculated as the mean of the standards for the state assessments where "Level III" was associated the "college and career ready" performance level for the state. Next, the functions for each performance level were smoothed using a cubic equation. Finally, the functions for each performance level were used to extrapolate down to Kindergarten and Grades 1 and 2 and up to Grade 12 and then hand-smoothed where necessary.

The final performance levels for each grade used with the LevelSet Math assessments are presented in *Table 14*.

Table 14. Revised Achieve3000 Math 4-level performance standards in the Quantile metric.

Grade	Level I	Level II	Level III	Level IV
K	EM120Q & Below	EM115Q to 35Q	40Q to 260Q	265Q & Above
1	105Q & Below	110Q to 255Q	260Q to 450Q	455Q & Above
2	280Q & Below	285Q to 440Q	445Q to 630Q	635Q & Above
3	440Q & Below	445Q to 595Q	600Q to 805Q	810Q & Above
4	545Q & Below	550Q to 720Q	725Q to 935Q	940Q & Above
5	640Q & Below	645Q to 830Q	835Q to 1030Q	1035Q & Above
6	730Q & Below	735Q to 915Q	920Q to 1100Q	1105Q & Above
7	815Q & Below	820Q to 990Q	995Q to 1155Q	1160Q & Above
8	890Q & Below	895Q to 1050Q	1055Q to 1205Q	1210Q & Above
9	965Q & Below	970Q to 1105Q	1110Q to 1255Q	1260Q & Above
10	1025Q & Below	1030Q to 1155Q	1160Q to 1310Q	1315Q & Above
11	1065Q & Below	1070Q to 1200Q	1205Q to 1390Q	1395Q & Above
12	1085Q & Below	1090Q to 1230Q	1235Q to 1425Q	1430Q & Above

Using LevelSet Math Assessment Results

In this era of student-level accountability and high-stakes assessment, differentiated instruction—the attempt "on the part of classroom teachers to meet students where they are in the learning process and move them along as quickly and as far as possible in the context of a mixed-ability classroom" (Tomlinson, 1999)—is a means for all educators to help students succeed. Differentiated instruction promotes high-level and powerful curriculum for all students, but varies the level of teacher support, task complexity, pacing, and avenues to learning based on student readiness, interest, and learning profile.

One strategy for managing a differentiated classroom suggested by Tomlinson is the use of multiple resources and supplementary materials that can be identified with the aid of the Quantile Framework. Equipped with a student's Quantile measure, teachers can connect him or her to textbook lessons, worksheets, games, websites, and trade books that have appropriate Quantile measures (Smith, no date; Smith and Turner, 2012). By incorporating Quantile measures into the planning of mathematics instruction, it becomes possible to forecast with greater probability how successfully students are likely to understand the material presented to them. Teachers can provide instruction on QSCs with Quantile measures below the targeted instruction when students are not ready for that instruction by focusing on prerequisite QSCs. On the other hand, teachers can focus enrichment activities on the impending QSCs.

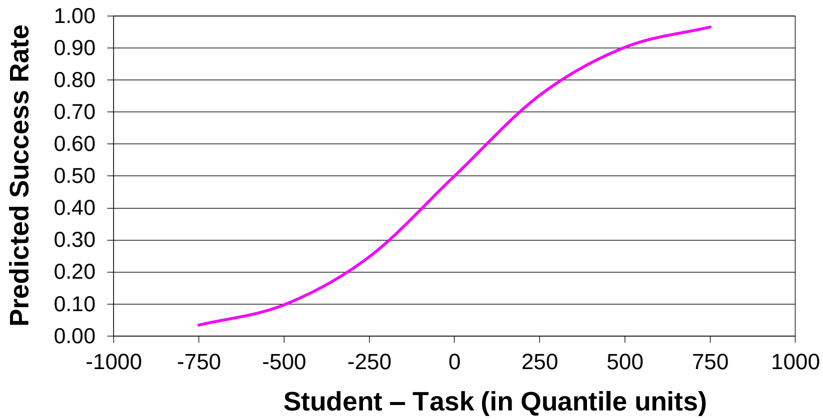
Forecasting Student Understanding and Predicting Success Rates

A student with a Quantile measure of 600Q who is to be instructed on mathematical tasks calibrated at 600Q is expected to have a 50% success rate on the tasks and a 50% understanding rate of the skills and concepts. This 50% rate is the basis for selecting tasks employing skills and concepts for instruction targeted to the student's mathematical achievement. If the mathematical demand of a task is less than the student measure, the success rate will exceed 50%. If the mathematical demand is much less, the success rate will be much greater. The difference in Quantile scale units between student achievement and mathematical demand governs understanding and success. This section gives more explicit information on forecasting student understanding and predicting success rates.

If all of the tasks associated with a 400Q Quantile Skill and Concept had the same difficulty, the understanding rate resulting from the 200Q difference between the 600Q student and the 400Q mathematical demand could be determined using the Rasch model equation (see Equation 3). This equation describes the relationship between the measure of a student's level of mathematical understanding and the difficulty of the skills and concepts. The average difficulty level of the tasks *and* their variability both affect the success rate. While understanding rates calculated only using this procedure would be biased because the difficulties of the tasks associated with a skill or concept are not all the same, the general relationship can be informative.

Figure 9 shows the general relationship between student-task discrepancy and predicted success rate. When the student measure and the task mathematical demand are the same, then the predicted success rate is 50% and the student is ready for instruction on the skill or concept. If a student has a measure of 600Q and the task's mathematical demand is 400Q, the difference is 200Q. According to *Figure 9*, a difference of +200Q (student measure minus task difficulty) indicates a predicted success rate of approximately 75%. Also note that a difference of –200Q indicates a predicted success rate of about 25%.

Figure 9. Student-mathematical demand discrepancy and predicted success rate.



The subjective experience between 25%, 50%, and 75% understanding or success varies greatly:

- A 1000Q student being instructed on 1000Q QSCs (50% forecasted understanding) will likely have a successful instructional experience—he has the background knowledge needed to learn and apply the new information. Teachers working with such a student report that the student can engage with the skills and concepts that are the focus of the instruction and, as a result of the instruction, are able to solve problems utilizing those skills. In short, such students appear to understand what they are learning.
- A 1000Q student being instructed on 1200Q QSCs (25% forecasted understanding) encounters so many unfamiliar skills and difficult concepts that the learning is frequently lost. Such students report frustration and seldom engage in instruction at this level of understanding.
- A 1000Q student being instructed on 800Q QSCs (75% forecasted understanding) reports being able to engage with the skills and concepts with minimal instruction, is able to solve complex problems related to the skills and concepts, is able to connect the skills and concepts with skills and concepts from other strands, and experiences automaticity of skills.

Table 15 gives an example of the forecasted understanding (or likely success rates) for specific skills for a specific student. *Table 16* shows forecasted understanding for one specific skill calculated for different student achievement measures.

Table 15. Success rates for a student with a Quantile measure of 750Q and skills of varying difficulty (demand).

Student Mathematics Achievement	Skill Demand	Skill Description	Forecasted Understanding
750Q	350Q	Locate points on a number line.	90%
750Q	550Q	Use order of operations, including parentheses, to simplify numerical expressions.	75%
750Q	750Q	Translate between models or verbal phrases and algebraic expressions.	50%
750Q	950Q	Estimate and calculate areas with scale drawings and maps.	25%
750Q	1150Q	Recognize and apply definitions and theorems of angles formed when a transversal intersects parallel lines.	10%

Table 16. Success rates for students with different Quantile measures of achievement for a task with a Quantile measure of 850Q.

Student Mathematics Achievement	Problems Related to "Locate points in all quadrants of the coordinate plane using ordered pairs."	Forecasted Understanding
450Q	850Q	10%
650Q	850Q	25%
850Q	850Q	50%
1050Q	850Q	75%
1250Q	850Q	90%

The primary utility of the Quantile Framework is its ability to forecast what happens when students engage with specific mathematical skills and concepts. The Quantile Framework makes a point forecast every time a skill is chosen for a student. There is error in skill measures, student measures, and their difference modeled as forecasted understanding rates (or predicted success rates on a specific task). However, the error is sufficiently small that the judgments about the students, task demands, and forecasted rates are useful.

Limitations of Achieve3000 Math and the Quantile Framework

Just as variables other than temperature affect comfort, variables other than content demand affect mathematical understanding. A student's background knowledge is known to affect understanding. However, although temperature alone does not fully identify the comfort level of an environment, we do not dismiss the importance of the information communicated by temperature. Similarly, the information communicated by the Quantile Framework is valuable, even though other information also enhances instructional decisions. In fact, the meaningful communication that is possible when test results are linked to instruction provides the opportunity for parents and students to give input regarding interests and background knowledge.

Results of LevelSet Math Assessments and Grade Levels. Quantile measures do not translate specifically to grade levels. Within any grade, there will be a range of student abilities and a range of concepts and skills for instruction. In a first-grade classroom there will be some students who are far ahead of the others and there will be some students who are behind the others in terms of mathematical achievement. To say that some skills and concepts are “just right” for first graders assumes that all first graders are learning at the same level. The Quantile Framework can be used to match students with mathematical skills and concepts at whatever level the student is working.

Simply because a student is very successful in mathematics, it should not be assumed that the student would necessarily understand material typically found at a higher grade level. Without adequate background knowledge, the skills and concepts may not have sufficient meaning to the student. A high Quantile measure for a grade indicates that the student can understand grade-appropriate mathematical skills and concepts at a higher understanding level (90%, for example).

The real power of the Quantile Framework is in examining the growth of students—wherever the student may be in the development of his or her mathematical skills. Students can be matched with skills and concepts where they are forecasted to understand 50% of the content prior to instruction. As a student grows, he or she can be matched with more demanding skills and concepts. And, as the skills and concepts become more demanding, the student grows.

The Quantile Item Bank

The Quantile Item Bank is a proprietary collection of original MetaMetrics' assessment items available for licensing by partners such as Achieve3000, Inc. The items in this bank have been written to assess a large range of mathematics ability. Experienced MetaMetrics' Editorial, Content, and Research staff have developed the items, including graphics and stems, adhering to clear guidelines described below. The items have been reviewed throughout the process to ensure the highest quality possible and, where possible, field tested to empirically examine the difficulty of the items.

When licensing items from the Quantile Item Bank, test publishers work with MetaMetrics to determine the appropriate specifications of the each test based on student population, programming focus, and other factors unique to the partner's organization. Then the tests are created by selecting the best items from the Quantile Item Bank to fit the specifications.

Item Writing

Item writers were provided with training materials concerning the development of multiple-choice items and the Quantile Framework for Mathematics. Item writers were classroom teachers and other educators who have had experience with the everyday mathematics ability and instruction of students at various levels. The use of individuals with these types of experiences helped to ensure that the items are valid measures of mathematics ability.

Item writers were also provided training related to sensitivity issues. Part of the item writing materials addressed these issues and identified areas to avoid when developing items. These materials were developed based on material published on universal design and fair-access—equal treatment of the sexes, fair representation of minority groups, and fair representation of disabled individuals. These guidelines were developed based on material published by CTB/McGraw-Hill (*Guidelines for Bias-Free Publishing*) and were assembled from the results of MetaMetrics' collaboration with various partners in textbook and test publishing.

1. Violence/crime: Avoid weapons, fights, arrests, illegal activities, abuse, and murders.
2. Depressing situations or death: Avoid sickness, death, and other negative situations.
3. Offensive language: Avoid use of curse words or words used to cover up a harsher curse; avoid oaths such as "Oh God!", words that belittle others or other insulting words such as "backwards," "ugly."
4. Drugs/alcohol/tobacco: Avoid any mention of drugs, alcohol, tobacco and anything associated with these topics such as rehab, bars, etc.
5. Sex/attraction: Avoid issues that call for a discussion of sex, sexual orientation, or relationships of either a romantic or sexual nature.
6. Race: Avoid racial slurs, belittling words, stereotypes (e.g., referring to Native Americans as Indians) and unbalanced representations of a race (e.g., mentioning African Americans only in the context of slavery).
7. Class: Avoid mentioning economic and social differences and avoid stereotypes.

8. Gender: Use gender free language (e.g., firefighter instead of fireman); avoid using male pronouns to refer to both sexes; show both genders in a variety of roles; avoid stereotypical portrayals of men or women.
9. Religion: Avoid selections that promote or demean a religious belief; avoid the assumption that people share a common belief; avoid mention of a reference to any holidays of a religious nature (e.g., Christmas, Halloween).
10. Supernatural/magic: Avoid mention of witches, goblins, wizards, and other supernatural beings; avoid magic in general.
11. Parents/family: Avoid selections that question parents, authority, or judgment; avoid negative relationships within the family; avoid raising the issue of alternative families.
12. Politics: Avoid controversial issues (e.g., unions, strikes) and selections, which portray political bias.
13. Animals/environment: Avoid hunting and cruelty to animals (e.g., fur coats, trapping animals) and be sensitive to environmental issues and animal rights.
14. Brand names/junk food: Avoid mentioning either.

Item writers were directed to have a range of thinking skills across the items developed. Excerpts on "mathematical complexity" from the NAEP 2005 Mathematics Assessment Framework were distributed to the items writers.

1. Low complexity items rely on recall and recognition of previously learned concepts and principles.
2. Moderate complexity items ask the student to apply knowledge and skills in new situations and to analyze what is happening in the problem.
3. High complexity items ask the student to reason about what might be true, to integrate prior knowledge and skills with the information presented in the problem, and to evaluate solutions based on specified criteria.

Item writers were also provided with a MetaMetrics style guide explaining the format, special characters and symbols, and specific stylistic issues. Item writers were provided information related to student use of ancillary materials: calculator (model, specifications), ruler, protractor, etc.

Item writers were required to write rationales for each distractor. Each item contains four responses (A, B, C, or D). Three of the responses are considered foils or distractors and one, and only one, response is the correct or best answer. Items were written so that the foils represent typical errors, misconceptions, or miscalculations. Item writers were encouraged to write foils based on their own classroom experiences and/or common error patterns documented in texts such as R. B. Ashlock's book *Error Patterns in Computation* (2010).

In keeping with the findings and recommendation of the National Math Panel (National Mathematics Advisory Panel, 2008), item writers developed items with minimal "nonmathematical sources of influence on student performance" (p. xxv). Unnecessary context was avoided where possible and anything that could be considered culturally or economically biased was removed.

Item Review

Each item was reviewed when first submitted according to the following criteria:

- *Content*: Each item was reviewed and/or edited by multiple trained mathematics educators to determine whether the item was aligned with the content Quantile Framework QSC that it was written to measure.
- *Bias*: Multiple reviewers examined each item to see if there was any indication of bias. Reviewers were provided the same information as the item writers concerning universal design and fair access.
- *Sensitivity*: The following areas were covered in the item writing materials: violence and crime, depressing situations/death, offensive language, drugs/alcohol/tobacco, sex/attraction, class, religion, supernatural/magic, parent/family, politics, animals/environment, and brand names/junk food.

Items were then reviewed and edited by a group of specialists that represented various perspectives—test developers, editors, and curriculum specialists. These individuals examined each item to ensure that it was a valid representation of the QSC for which it was written, for sensitivity issues, and for the quality of the response options. The following criteria were used to review each item:

- Grade Appropriate:
  - Are the skills required appropriate to the grade level?
  - Is the context appropriate to the grade level?
- Stem:
  - Is the stem concise, well-written, and free of grammatical errors?
  - Does the question tell students what they are expected to do?
  - Does the question focus on important mathematics?
  - Is the question realistic and mathematically sound?
  - Is the context realistic?
- Key:
  - Is there a unique key and is it indicated correctly?
  - Do the other answer choices represent common errors?
  - Are the answer choices ordered in a logical way?
- Art Specifications:
  - Is the art an integral part of the item?
  - Is the art accurate and free of misinterpretation?
  - Are art specs written clearly with detailed information for artists?
- Content Mapping:
  - Does the item assess the strand and QSC it purports to measure?
  - Does the item assess the cognitive skill it purports to measure?
- Readability:
  - Are the language, mathematical terminology, and overall readability appropriate for the grade level?
- Computation:
  - Are the computation skills necessary to work the problem appropriate for the grade level?

- Independence:
  - Can the item be solved independent of all other items in the set?
  - Would any of the other items in the set help students solve this question?
- Name/Bias:
  - Is the name recognizable and free of misinterpretation?
  - Could the item cause examinees to become upset, distracted, or less motivated?
  - Is the item free of culturally specific language?
  - Are all groups portrayed accurately and fairly, avoiding stereotypes and stereotypical roles?
  - If background knowledge is expected, is it familiar to all ethnic and geographic groups?
- Format/Style/Metadata:
  - Are the questions formatted according to the style guide?
  - Are tables formatted according to the style guide?
  - Are the correct symbols used?
  - Are the item and art charts/templates filled out completely and correctly?

Quantile Mathematics Ability Item Bank Specifications

In August 2020 the Quantile Item Bank consisted of 4,322 items.

Each item in the Quantile Item Bank is associated with one of the six content strands of the Quantile Framework and the proportion of items by grade and strand is shown in *Table 17*.

1. Number Sense (NS)
2. Geometry (G)
3. Algebra and Algebraic Thinking (A)
4. Data Analysis, Statistics & Probability (DA)
5. Measurement (M)
6. Numerical Operations (NO)

Each item is also associated with one QSC within the Quantile Framework and information related to Webb's Depth of Knowledge (DOK Level) and use of tools is also assigned to each item. The difficulty of each item is based on the Quantile Theory and each item received a theoretical calibration based on the associated QSC (for more information, see the section on "Quantile Skill and Concept (QSC) Measures" on pages 24 through 26 earlier in this development and technical guide). The mean and range of item difficulty by grade is shown in *Table 18*.

Table 17. Quantile Item Bank item content by grade and strand.

Grade/Course	*N* items	Percent of Grade-Level Items by Strand					
		NS	G	A	DA	M	NO
K	375	33.6	17.3	15.5	7.2	10.4	16.0
1	407	30.0	11.3	13.0	7.1	9.8	28.7
2	561	20.9	8.7	16.2	5.7	9.3	39.2
3	526	18.8	6.8	20.3	7.6	13.1	33.3
4	473	23.3	8.7	11.4	5.3	11.4	40.0
5	326	21.8	7.7	11.3	4.9	8.9	42.3
6	314	17.5	6.1	32.2	15.0	8.0	21.3
7	277	9.7	7.6	29.2	18.1	10.5	24.9
8	282	11.0	11.0	47.9	8.9	4.6	16.7
A1/M1	370	0.0	0.2	74.6	15.1	7.3	2.7
G/M2	357	0.0	61.9	9.8	12.0	16.2	0.0
A2/M3	54	13.0	5.6	57.4	3.7	7.4	13.0

Table 18. Quantile Item Bank item difficulty (Quantile measure) descriptive statistics, by grade.

Grade/Course	*N* items	Quantile measure Mean (SD)	Quantile measure Range
K	375	-92Q (106.94)	-310Q to 250Q
1	407	-13Q (104.01)	-310Q to 300Q
2	561	136Q (159.41)	-150Q to 530Q
3	526	305Q (165.94)	-130Q to 680Q
4	473	493Q (196.12)	-80Q to 920Q
5	326	626Q (161.31)	150Q to 930Q
6	314	756Q (154.83)	370Q to 1070Q
7	277	852Q (142.23)	400Q to 1100Q
8	282	955Q (129.49)	690Q to 1250Q
A1/M1	370	1073Q (148.34)	520Q to 1400Q
G/M2	357	1097Q (166.62)	400Q to 1360Q
A2/M3	54	1175Q (133.98)	810Q to 1400Q

Quantile Item Bank Field Testing and Calibration

In addition to content and sensitivity reviews during the development process, Quantile Item Bank items are field-tested as part of MetaMetrics on-going research. Quantile Item Bank items may be field-tested as part of stand-alone research field tests or they may be embedded within research tests for concurrent projects. Several recent research studies including Quantile Item Bank items are described below.

Field-Test Analyses

During these field studies, Quantile Item Bank items were analyzed using both the classical measurement model and the Rasch (one-parameter logistic item response theory) model. Item statistics and descriptive information (item number, field test form and item position, and answer key) were compiled for each item

Classical Measurement. For each item, the p-value (percent correct) and the point-biserial correlation between the item score (correct response) and the total test score were computed. Point-biserial correlations were also computed between each of the incorrect responses and the total score. In addition, frequency distributions of the response choices (including omits) were tabulated (both actual counts and percents). *Table 20* displays the classical item statistics.

Rasch Item Response Theory. Classical test theory has two basic shortcomings: (1) the use of item indices whose values depend on the particular group of examinees from which they were obtained, and (2) the use of examinee ability estimates that depend on the particular choice of items selected for a test. The basic premises of item response theory (IRT) overcome these shortcomings by predicting the performance of an examinee on a test item based on a set of underlying abilities (Hambleton and Swaminathan, 1985). The relationship between an examinee's item performance and the set of traits underlying item performance can be described by a monotonically increasing function called an item characteristic curve (ICC). This function specifies that as the level of the trait increases, the probability of a correct response to an item increases.

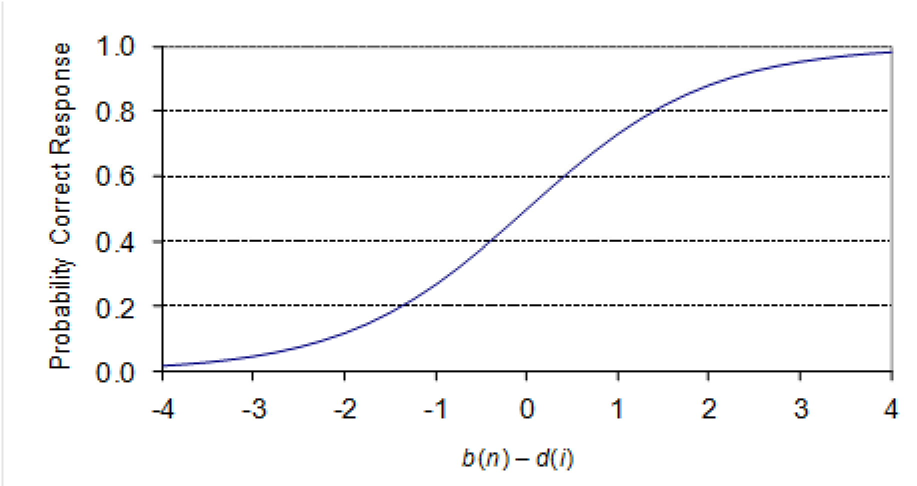
The conversion of observations into measures can be accomplished using the Rasch (1980) model, which states a requirement for the way that item calibrations and observations (count of correct items) interact in a probability model to produce measures. The Rasch IRT model expresses the probability that a person (*n*) answers a certain item (*i*) correctly by the following relationship:

$$P_{ni} = \frac{e^{b_n - d_i}}{1 + e^{b_n - d_i}} \qquad \text{(Equation 6)}$$

where d_i is the difficulty of item *i* ($i = 1, 2, \ldots$, number of items);
b_n is the ability of person *n* ($n = 1, 2, \ldots$, number of persons);
$b_n - d_i$ is the difference between the ability of person *n* and the difficulty of item *i*; and
P_{ni} is the probability that examinee *n* responds correctly to item *i*
(Hambleton and Swaminathan, 1985; Wright and Linacre, 1994).

This measurement model assumes that item difficulty is the only item characteristic that influences the examinee's performance such that all items are equally discriminating in their ability to identify low-achieving persons and high achieving persons (Bond and Fox, 2001; Hambleton, Swaminathan, and Rogers, 1991). In addition, the lower asymptote is zero, which specifies that examinees of very low ability have zero probability of correctly answering the item. The Rasch model has the following assumptions: (1) unidimensionality—only one ability is assessed by the set of items; and (2) local independence—when abilities influencing test performance are held constant, an examinee's responses to any pair of items are statistically independent (conditional independence, i.e., the only reason an examinee scores similarly on several items is because of his or her ability, not because the items are correlated). The Rasch model is based on fairly restrictive assumptions, but it is appropriate for criterion-referenced assessments. *Figure 10* graphically shows the probability that a person will respond correctly to an item as a function of the difference between a person's ability and an item's difficulty.

Figure 10. The Rasch Model--the probability person n responds correctly to item i.



An assumption of the Rasch model is that the probability of a response to an item is governed by the difference between the item calibration (d_i) and the person's measure (b_n). From an examination of the graph in *Figure 10*, when the ability of the person matches the difficulty of the item ($b_n - d_i = 0$), then the person has a 50% probability of responding to the item correctly.

The number of correct responses for a person is the probability of a correct response summed over the number of items. When the measure of a person greatly exceeds the calibration (difficulties) of the items ($b_n - d_i > 0$), then the expected probabilities will be high and the sum of these probabilities will yield an expectation of a high "number correct." Conversely, when the item calibrations generally exceed the person measure ($b_n - d_i < 0$), the modeled probabilities of a correct response will be low and the expectation will be a low "number correct."

Thus, Equation 6 can be rewritten in terms of the number of correct responses of a person on a test

$$O_p = \sum_{i=1}^{L} \frac{e^{b_n - d_i}}{1 + e^{b_n - d_i}} \qquad \text{(Equation 7)}$$

where O_p is the number of correct responses of person p and L is the number of items on the test. When the sum of the correct responses and the item calibrations (d_i) is known, an iterative procedure can be used to find the person measure (b_n) that will make the sum of the modeled probabilities most similar to the number of correct responses. One of the key features of the Rasch IRT model is its ability to place both persons and items on the same scale. It is possible to predict the odds of two individuals being successful on an item based on knowledge of the relationship between the abilities of the two individuals. If one person has an ability measure that is twice as high as that of another person (as measured by *b*—the ability scale), then he or she has twice the odds of successfully answering the item.

Equation 7 possesses several distinguishing characteristics:

- The key terms from the definition of measurement are placed in a precise relationship to one another.
- The individual responses of a person to each item on an instrument are absent from the equation. The only information that appears is the "count correct" (O_p), thus confirming that the raw score (i.e., number of correct responses) is "sufficient" for estimating the measure.

For any set of items the possible raw scores are known. When it is possible to know the item calibrations (either theoretically or empirically from field studies), the only parameter that must be estimated in Equation 7 is the person measure that corresponds to each observable count correct. Thus, when the calibrations (d_i) are known, a correspondence table linking observation and measure can be constructed without reference to data on other individuals.

The item responses were submitted to a Winsteps IRT analysis. The resulting item difficulties (in logits) were assigned Quantile measures by multiplying by 180 and anchoring each set of items to the mean theoretical difficulty of the items on the form. *Table 20* presents the item response theory results (Quantile measures).

Table 20. Item-level descriptive statistics of Quantile Item Bank items included in studies administered in the United States.

Study	Number of Items Tested	Number of Student Responses Per Item	*p*-value Mean (SD)	Point-biserial Mean (Range)	Quantile Measure Mean (SD)
1-G7	33	133-272	0.35 (0.12)	0.29 (-0.03-0.51)	1071.56 (125.58)
1-G8	48	67-74	0.39 (0.16)	0.32 (-0.02-0.56)	1098.57 (152.66)

Where necessary, items are deleted from the item bank or revised and recalibrated. The item data from the field studies is also used to inform item selection from the Quantile Item Bank for future projects.

Development of the LevelSet Math Assessments

The LevelSet Math assessments were designed to indicate what mathematical skills and concepts the student is ready for next. For each grade level, one LevelSet Math assessment is to be administered which addresses the prior grade-level content and two additional LevelSet Math assessments are to be administered during the school year which address the current grade-level content.

In 2020, Achieve3000, Inc. identified the need for a mathematics assessment that would provide their students with a Quantile measure. Test specifications for the LevelSet Math assessments were developed in the winter of 2020 for Kindergarten through Grades 11/12. Items from the Quantile Item Bank were identified and reviewed from the winter of 2020 to the summer of 2020. Further information about the test development process is detailed in the following section.

LevelSet Math Assessment Specifications and Development

The specifications for the LevelSet Math assessment forms were developed to cover skills in pre-Kindergarten through Grades 11/12 that align with the Common Core State Standards for Mathematics (CCSS) (National Governors Association and Council of Chief State School Officers, 2010a, 2010b). Students who take the Beginning of Year LevelSet Math assessment for their grade level will be administered items that cover the Quantile Skills and Concepts (QSCs) associated with the alignment to the CCSS at the prior grade level.

Each level of the LevelSet Math assessment covers the following Quantile Framework content strands that have been aligned with the CCSS:

- Number Sense
- Numerical Operations
- Algebra and Algebraic Thinking
- Geometry
- Measurement
- Data Analysis, Statistics, and Probability

A central element of determining a student's Quantile measure is that the assessments must include items from a majority of the six content strands. This ensures the integration among the content strands into a single measure of mathematical ability.

Each QSC in the Quantile Framework is aligned with one of the six content strands. For the LevelSet Math assessment forms, the proportion of each content strand at each grade level is determined by the alignment of the Quantile Framework to the strand distribution of the CCSS curriculum as shown in *Tables 21* through *Table 23.*

Table 21. LevelSet Math BOY assessment strand distribution, by grade level.

BOY Form	K	G1	G2	G3	G4	G5
Number Sense	50%	60%	30%	25%	15%	30%
Geometry	25%	10%	5%	5%	5%	5%
Algebra and Algebraic Thinking	5%	10%	15%	10%	15%	15%
Data Analysis, Statistics and Probability	0%	0%	10%	10%	5%	5%
Measurement	0%	5%	15%	25%	20%	10%
Numerical Operations	20%	15%	25%	25%	40%	35%

BOY Form	G6	G7	G8	G9	G10	G11/12
Number Sense	15%	15%	10%	10%	3%	3%
Geometry	10%	10%	10%	15%	7%	53%
Algebra and Algebraic Thinking	5%	30%	20%	45%	53%	7%
Data Analysis, Statistics and Probability	10%	10%	15%	10%	20%	17%
Measurement	20%	10%	10%	10%	10%	17%
Numerical Operations	40%	25%	35%	10%	7%	3%

Table 22. LevelSet Math MOY assessment strand distribution, by grade level.

MOY Form	K	G1	G2	G3	G4	G5
Number Sense	60%	40%	35%	15%	30%	15%
Geometry	10%	10%	5%	5%	5%	10%
Algebra and Algebraic Thinking	10%	15%	15%	15%	15%	5%
Data Analysis, Statistics and Probability	0%	0%	10%	5%	5%	10%
Measurement	5%	10%	10%	20%	10%	20%
Numerical Operations	15%	25%	25%	40%	35%	40%

MOY Form	G6	G7	G8	G9	G10	G11/12
Number Sense	15%	10%	10%	7%	3%	3%
Geometry	10%	10%	10%	15%	35%	15%
Algebra and Algebraic Thinking	30%	25%	45%	55%	20%	45%
Data Analysis, Statistics and Probability	10%	10%	15%	10%	20%	17%
Measurement	10%	10%	10%	10%	15%	17%
Numerical Operations	25%	35%	10%	3%	7%	3%

Table 23. LevelSet Math EOY assessment strand distribution, by grade level.

EOY Form	K	G1	G2	G3	G4	G5
Number Sense	60%	30%	25%	15%	30%	15%
Geometry	10%	5%	5%	5%	5%	10%
Algebra and Algebraic Thinking	10%	15%	10%	15%	15%	5%
Data Analysis, Statistics and Probability	0%	10%	10%	5%	5%	10%
Measurement	5%	15%	25%	20%	10%	20%
Numerical Operations	15%	25%	25%	40%	35%	40%

EOY Form	**G6**	**G7**	**G8**	**G9**	**G10**	**G11/12**
Number Sense	15%	10%	10%	3%	3%	3%
Geometry	10%	10%	15%	7%	53%	3%
Algebra and Algebraic Thinking	30%	20%	45%	53%	7%	70%
Data Analysis, Statistics and Probability	10%	15%	10%	20%	17%	10%
Measurement	10%	10%	10%	10%	17%	7%
Numerical Operations	25%	35%	10%	7%	3%	7%

Grade-level test specifications were developed by examining the Quantile measures of the QSCs aligned with the CCSS Kindergarten through Algebra II/Integrated Math 3 standards (for more information, see the alignment in the Quantile Math Skills Database on the Quantile Framework website at https://hub.lexile.com/math-skills-database). *Table 24* presents the mean Quantile measure specified for each grade level of the LevelSet Math test forms. Unique items were selected for each LevelSet Math form to ensure that increases in performance (student Quantile measure) were due to actual learning of the instructional content and not to other external factors (e.g., recall of the items).

Table 24. LevelSet Math assessment difficulty, by grade level.

Grade	**BOY Form Mean**	**MOY Form Mean**	**EOY Form Mean**
Kindergarten	-205Q	-140Q	-50Q
1	-50Q	65Q	120Q
2	120Q	230Q	310Q
3	310Q	405Q	470Q
4	470Q	550Q	620Q
5	620Q	665Q	710Q
6	710Q	770Q	800Q
7	800Q	855Q	890Q
8	890Q	935Q	980Q
9	980Q	1020Q	1055Q
10	1055Q	1080Q	1115Q
11/12	1115Q	1140Q	1170Q

For each level of the LevelSet Math assessment, the percentage of items per strand for the operational test forms is presented in *Table 25* through *Table 27.*

Table 25. LevelSet Math BOY assessment operational strand distribution, by grade level.

BOY Form	K	G1	G2	G3	G4	G5
Number Sense	50%	47%	30%	23%	17%	30%
Geometry	23%	10%	7%	13%	7%	7%
Algebra and Algebraic Thinking	7%	13%	7%	10%	13%	13%
Data Analysis, Statistics and Probability	0%	0%	10%	7%	7%	3%
Measurement	0%	10%	13%	23%	20%	10%
Numerical Operations	20%	20%	33%	23%	37%	37%

BOY Form	G6	G7	G8	G9	G10	G11/12
Number Sense	17%	13%	10%	10%	3%	3%
Geometry	7%	7%	10%	13%	7%	53%
Algebra and Algebraic Thinking	7%	33%	20%	47%	53%	7%
Data Analysis, Statistics and Probability	10%	10%	13%	10%	20%	17%
Measurement	20%	13%	10%	10%	10%	17%
Numerical Operations	40%	23%	37%	10%	7%	3%

Table 26. LevelSet Math MOY assessment operational strand distribution, by grade level.

MOY Form	K	G1	G2	G3	G4	G5
Number Sense	57%	40%	40%	20%	30%	13%
Geometry	10%	10%	3%	7%	7%	10%
Algebra and Algebraic Thinking	10%	10%	13%	17%	10%	10%
Data Analysis, Statistics and Probability	0%	0%	7%	3%	7%	7%
Measurement	3%	13%	7%	17%	10%	20%
Numerical Operations	20%	27%	30%	37%	37%	40%

MOY Form	G6	G7	G8	G9	G10	G11/12
Number Sense	13%	10%	10%	7%	3%	3%
Geometry	10%	7%	7%	13%	37%	13%
Algebra and Algebraic Thinking	33%	27%	50%	57%	20%	47%
Data Analysis, Statistics and Probability	7%	13%	17%	10%	20%	17%
Measurement	10%	13%	10%	10%	13%	17%
Numerical Operations	27%	30%	7%	3%	7%	3%

Table 27. LevelSet Math EOY assessment operational strand distribution, by grade level.

EOY Form	K	G1	G2	G3	G4	G5
Number Sense	60%	27%	23%	20%	30%	13%
Geometry	10%	7%	7%	10%	13%	10%
Algebra and Algebraic Thinking	10%	17%	13%	10%	7%	7%
Data Analysis, Statistics and Probability	0%	10%	10%	7%	3%	7%
Measurement	3%	10%	23%	17%	17%	20%
Numerical Operations	17%	30%	23%	37%	30%	43%

EOY Form	G6	G7	G8	G9	G10	G11/12
Number Sense	17%	10%	13%	3%	3%	3%
Geometry	7%	10%	17%	7%	53%	3%
Algebra and Algebraic Thinking	30%	20%	43%	53%	7%	70%
Data Analysis, Statistics and Probability	10%	20%	10%	20%	17%	10%
Measurement	10%	10%	10%	10%	17%	7%
Numerical Operations	27%	30%	7%	7%	3%	7%

Tables 28 and *29* shows the final mean item difficulty and range of item difficulties on the LevelSet Math operational assessment forms for each grade. Note that in each grade level, the BOY form reflects the material of the previous grade level and thus has a lower mean Quantile measure.

Table 28. Descriptive statistics of the operational LevelSet Math assessments, by grade and test form.

Grade	BOY Form Mean (SD)	MOY Form Mean (SD)	EOY Form Mean (SD)
Kindergarten	-167.67Q (96.51)	-110.17Q (115.18)	-56.33Q (116.07)
1	-54.83Q (117.88)	57.67Q (116.37)	90.00 (117.47)
2	80.33Q (120.96)	195.00Q (112.12)	282.33Q (112.45)
3	291.67Q (118.64)	386.67Q (111.09)	440.00Q (122.78)
4	448.67Q (118.90)	531.00Q (143.54)	608.00Q (133.48)
5	580.67Q (142.17)	643.00Q (136.69)	678.33Q (142.95)
6	686.00Q (144.31)	765.67Q (109.47)	810.33Q (135.76)
7	814.00Q (121.39)	860.00Q (116.80)	873.00Q (124.21)
8	880.33Q (119.47)	940.67Q (131.09)	982.00Q (113.97)
9	1005.33Q (128.73)	1025.33Q (124.56)	1061.67Q (122.7)
10	1054.67Q (120.88)	1089.00Q (138.95)	1130.00Q (113.93)
11/12	1128.33Q (117.74)	1152.67Q (124.93)	1182.00Q (122.51)

Table 29. Range of the LevelSet Math assessment item difficulties, by grade and test form.

Grade	BOY Form Range	MOY Form Range	EOY Form Range
Kindergarten	-310Q to 80Q	-310Q to 160Q	-310Q to 160Q
1	-310Q to 160Q	-150Q to 300Q	-150Q to 300Q
2	-150Q to 280Q	-100Q to 450Q	110Q to 530Q
3	110Q to 530Q	180Q to 630Q	250Q to 680Q
4	240Q to 680Q	310Q to 790Q	390Q to 820Q
5	320Q to 850Q	400Q to 920Q	480Q to 920Q
6	470Q to 930Q	520Q to 920Q	480Q to 1040Q
7	470Q to 1050Q	600Q to 1040Q	610Q to 1100Q
8	690Q to 1100Q	610Q to 1190Q	780Q to 1250Q
9	780Q to 1250Q	800Q to 1250Q	810Q to 1290Q
10	810Q to 1290Q	880Q to 1350Q	880Q to 1350Q
11/12	880Q to 1350Q	880Q to 1390Q	990Q to 1380Q

The final review process for the LevelSet Math assessment forms was conducted in a two-stage process. First, the test specifications were reviewed and distributions of correct responses were examined. Next, the tests were taken to verify the answer keys and review the foils in relation to the items. Finally, the overall test forms were reviewed for flow and consistency. Because the Quantile Theory was used to estimate the difficulties of the items, it was necessary for the item and test specifications to be adhered to as closely as possible.

Curricular Perspective

- Does the focus of each item (strand and QSC) in each form flow well?
- Are there any items that overlap in content or provide clues to other items?

Psychometric Perspective

- Do the final forms have the same approximate mean and range of Quantile measures as the target specifications?
- Is the distribution of the placement of correct answers within a form approximately equal (about 25% for each response position)?
- Are runs of the same correct response position avoided? (e.g., more than 3 of any response positions in a row would be undesirable.)

Scoring and Reporting

The main purposes of the LevelSet Math assessments are to measure student mathematical ability at the beginning of the school year, monitor progress during the school year, and connect assessment results with classroom instruction. In order to meet these goals, a developmental scale must be used to report the results. This section describes the statistical procedures and the analyses used to score and scale the LevelSet Math assessment results.

Scaling of the LevelSet Math Assessments

The LevelSet Math assessment scores are reported on the Quantile scale. The Quantile Framework spans the developmental continuum from kindergarten mathematics through the content typically taught in Algebra II, Geometry, Trigonometry, Precalculus, and Calculus, from below 0Q (Emerging Mathematician) to above 1600Q. Student Quantile measures are reported in 5-unit intervals. Measures with an EM represent Quantile measures below 0Q.

There are many reasons to use scale scores rather than raw scores to report test results. Scale scores overcome the disadvantage of many other types of scores (e.g., percentiles and raw scores), in that equal differences between scale score points represent equal differences in ability. Each question on a test has a unique level of difficulty; therefore, answering 17 questions correctly on one form of a test requires a slightly different level of ability from answering 17 items correctly on another form of the test. But, receiving a scale score (Quantile measure) of 175Q on one form of a test represents a similar level of mathematical ability as receiving a scale score (Quantile measure) of 175Q on another form of the test.

Individual scores are calculated by first summing the number of correct responses (omitted items are counted as incorrect) on the test. The number correct is then converted to a scaled Quantile measure. This Quantile measure is recorded as the student's current Quantile measure.

Quantile measures and associated uncertainty estimates are calculated for each raw score by accessing a convenient cloud-based scoring service. Users of the scoring service send raw score assessment results (i.e., count correct and item identifiers) and receive a Quantile measure and the uncertainty of the measure in the Quantile metric.

Scoring LevelSet Math Assessments: The Bayesian Paradigm

Bayesian methodology provides a paradigm for combining prior information with current data, both of which are subject to uncertainty, and for arriving at an estimate of current status, which is again subject to uncertainty. Uncertainty is modeled mathematically using probability.

In the LevelSet Math assessment context, when a student is administered the Beginning of Year test form, the results from the test become the prior information for the following test administration—Middle of Year or End of Year test form. Each subsequent assessment uses prior information from all previous assessments. The current data in this context is the performance on the current test, which can be summarized as the number of items answered correctly out of the total number of items attempted.

Both prior information and current data are represented via probability models reflecting uncertainty. The need for incorporating uncertainty when modeling prior information is intuitively clear. The need for incorporating uncertainty when modeling test performance is, perhaps, less intuitive. Once the test has been taken and scored, and assuming that no scoring errors were made, the performance, i.e. raw score, is known with certainty. Uncertainty arises because test performance is associated with, but not determined by, the ability of the student, and it is that ability, rather than the test performance per se, that we are endeavoring to measure. Any single performance may over- or underestimate a student's ability, based on factors such as luck, prior knowledge, etc. Thus, although we are certain about the test performance once the results have been calculated, we remain uncertain about the ability that produced the performance.

The uncertainty associated with prior knowledge is modeled by a probability distribution for the ability parameter. This distribution is called the prior distribution and it is usually represented by a probability density function (e.g., the normal bell-shaped curve). The uncertainty arising from current data is modeled by a probability function for the data when the ability parameter is held fixed. When roles are reversed so that the data are held fixed and the ability parameter is allowed to vary, this function is called the likelihood function. In the Bayesian paradigm, the posterior probability density for the ability parameter is proportional to the product of the prior density and the likelihood, and this posterior density is used to obtain the new ability estimate along with its uncertainty.

Modeling Growth and Its Impact on the Prior. Once a posterior has been obtained from current data, that posterior can serve as the prior for an immediate repeat assessment. If a substantial amount of time has passed since the last assessment, however, then allowance should be made for an uncertain amount of growth since the last assessment. This allowance is accomplished by means of a growth model, which estimates as a function of elapsed time both student growth and the augmentation in uncertainty.

A student's rate of growth in ability is variable. MetaMetrics has developed a growth-rate model based on an analysis of a longitudinal dataset that examined growth in reading and mathematics across grades 1 through 12 for approximately 100,000 students. The purpose of the study was to describe the functional form of growth across the grades during the school year. It was found that younger students grow at a faster rate than older, experienced students. Modeling the growth rate as a decreasing function of current ability (b_{old}) incorporated this difference. The resulting values in the equation were based on a regression analysis of the longitudinal dataset where growth was regressed on ability in order to determine the intercept (estimated average growth per year) and slope (change in growth per unit-change in ability). It was also found that by limiting the time period of interest to the school year, growth during this brief period could be treated as linear in that students make about the same amount of growth each month.

The data set to examine growth was collected from a mid-Atlantic school district served approximately 206,000 students. Eighty percent of the students served were racial/ethnic minorities, with 65% African American, 11% Hispanic, and 4% Asian American. According to the District Profile, 76.4% of the students were classified as "Low Income." There were 258 schools organized into 22 clusters, with each cluster consisting of a high school and its feeder schools (about 8 to 16 schools per cluster). Approximately 16% of the students were enrolled in special education programs, approximately 5% of the students were enrolled in gifted education programs, and approximately 5% of the students were enrolled in limited-English proficient programs. From the results of the research, the following conclusions were made: (1) race/ethnicity had a relatively small effect, and (2) all ethnic/racial groups were progressing at the same rate.

Scoring Process: Overview of Flow

1. *Administer initial LevelSet Math assessment*. During the initial (first) administration of a LevelSet Math assessment, students will receive a raw score which is converted to a Quantile measure based on his/her performance (using the MetaMetrics' Scoring Service API). The information from the LevelSet Math assessment (Quantile measure, uncertainty) becomes the prior information used by the Bayesian Scoring algorithm to calculate subsequent updated Quantile student measures.

2. *Administer another LevelSet Math assessment and compute new values.* This step uses the information from student performance on another administration of LevelSet Math (i.e., Middle of Year or End of Year forms) to produce a posterior density. This value is used to create the new Quantile measure and associated uncertainty for the student. The new Quantile measure and uncertainty for the student will be incorporated into the prior information for the scoring of subsequent tests. For each subsequent administration of a LevelSet Math assessment, all of the information on the student's mathematics ability from the previous test administrations is incorporated into the student's prior.

Reporting

Conventions for Reporting. The Quantile Framework spans the developmental continuum from prekindergarten mathematics through the content typically taught in Algebra II, Geometry, Trigonometry, Precalculus, and Calculus, from below 0Q (Emerging Mathematician) to above 1600Q. Quantile measures of one thousand or greater are reported without a comma (e.g., 1050Q). All student Quantile measures are rounded to the nearest 5Q. Measures below 0Q are reported as EMxxxQ (e.g., a Quantile measure of -120 is reported as EM120Q) where "EM" stands for "Emerging Mathematician" and replaces the negative sign in the number. As with any test score, uncertainty is present in the form of measurement error.

The Quantile scale is like a thermometer, with numbers below zero indicating decreasing mathematical achievement as the number moves away from zero. The smaller the number following the EM code, the more advanced the student is. For example, an EM150Q student is more advanced than an EM200Q student. Above 0Q, measures indicate increasing mathematical achievement as the numbers increase. For example, a 200Q student is more advanced than a 150Q student. The lowest reported value below 0Q is EM400Q.

The results from the LevelSet Math assessments can be used to match students with instructional resources to provide a successful experience for each student. When the purpose of assessment is to match students with instructional resources, Quantile measures below 0Q are capped on the lower end at EM400Q. Also, when a student scores at the upper end of the distribution (typically greater than 90% correct), then the assessment may be mistargeted and provides little or no actionable information related to instruction. As described in the scoring service documentation, the Quantile measures associated with each level of the LevelSet Math assessment should be capped at the upper end of the distribution at the Quantile measures associated with the 90^{th} percentile of performance based on national linking studies conducted by MetaMetrics. MetaMetrics expresses these measures used for instructional purposes as “Reported Quantile Measures” and recommends that they be used on individual score reports.

Test Administration and Use Guidelines

Assessment practices should be in accordance with the generally accepted ethical standards of the education profession. Accordingly, any practice that increases students' scores should simultaneously represent an increase in students’ mastery (i.e., increasing students’ abilities to perform skills or demonstrate knowledge in real world situations) of the content domains tested. For more information, refer to *Standards for Educational and Psychological Testing* (AERA, APA, NCME, 2014).

Students should not be administered a specific test form more than once within any one year. When a student takes the same form for a second time within the span of one year, we are unsure as to how to interpret a change in Quantile measure because bias may have been introduced due to exposure to the assessment materials. Was the change because the student’s mathematical achievement has improved/grown, or was the change because the student remembers some of the items and has experience with the testing environment? Furthermore, items from the LevelSet Math assessments should not be used as part of instruction.

The Quantile measures reported from the LevelSet Math assessments should be used to inform instruction and interpreted within the context of the LevelSet Math administration. The Quantile measures can be used by educators, students, and parents to improve instruction and learning by matching students with appropriate resources given their mathematics level. Examples of appropriate uses of Quantile measures from LevelSet Math:

- The Quantile measures can be used to help explain students’ assessment results for educators, students and parents by describing the likely range of what students know and are able to do through a connection to instructional materials and activities.
- Educators, students, and parents may use the Quantile links to instructional resources and activities that can serve as a guide to help students stretch to a higher level to improve their understanding of the content.
- Quantile measures can be used to help interpret progress on the LevelSet Math by describing the nature and potential implications of achievement gaps, the gains necessary for students to reach the requisite level of college and career readiness, and instructional materials that may be helpful in accelerating students’ learning.

Analysis of individual test item results should only be done in the context of the item, the instruction, and the group of students administered the assessment. For example, it should not be concluded that students answering a particular item correctly (item seems easy) implies the student's Quantile measure should be higher. Scale scores (Quantile measures) are used to scale item difficulties relative to each other, easier items have lower Quantile item difficulties and harder items have higher Quantile item difficulties. The use of scale scores allows for the comparability of scores across different forms of the assessment consisting of different sets of items.

Reliability

If use is to be made of some piece of information, then the information should be reliable—stable, consistent, and dependable. In reality, all test scores have some error (or level of uncertainty). This uncertainty in the measurement process is related to three factors: (1) the statistical model that was used to compute the score, (2) the items that were used to determine the score, and (3) the condition of the student when the items used to determine the score were collected. Once the level of uncertainty in a test score is known then it can be taken into account when using the test results.

Reliability, or the consistency of scores obtained from an assessment, is a major consideration in evaluating any assessment procedure. Uncertainty has been examined with the LevelSet Math assessments in terms of examinee error and QSC measure error.

Uncertainty and Standard Error of Measurement

Because of the presence of measurement error associated with test unreliability, there is always some uncertainty about a student's true score. This uncertainty is known as the standard error of measurement (SEM). The magnitude of the SEM of an individual student's score depends on the following characteristics of the test:

- The number of test items—smaller standard errors are associated with longer tests.
- The quality of the test items—in general, smaller standard errors are associated with highly discriminating items for which correct answers cannot be obtained by guessing.
- The match between item difficulty and student ability—smaller standard errors are associated with tests composed of items with difficulties approximately equal to the ability of the student (targeted tests)

(Hambleton, Swaminathan, and Rogers, 1991).

Whenever a model is used to explain the relationship between parameters, some of the differences between observed and theoretical measures cannot be explained. The LevelSet Math assessments were developed using the Rasch one-parameter item response theory model to relate a student’s mathematical ability and the difficulty of the items. There is a unique amount of measurement error due to model misspecification (violation of model assumptions) associated with each score on the assessment. *Table 30* describes the uncertainties due to model misspecification for the LevelSet Math assessments.

Table 30. Uncertainties (25% - 75% correct) for the LevelSet Math assessments, by Quantile range.

Quantile Range	Kindergarten			Grade 1			Grade 2		
	BOY	**MOY**	**EOY**	**BOY**	**MOY**	**EOY**	**BOY**	**MOY**	**EOY**
EM600Q to EM505Q	74Q	77Q							
EM500Q to EM405Q	70Q	72Q	76Q	76Q					
EM400Q to EM305Q	68Q	69Q	70Q	71Q	75Q	77Q	77Q		
EM300Q to EM205Q	72Q	69Q	69Q	69Q	71Q	73Q	73Q		
EM200Q to EM105Q	78Q	74Q	71Q	71Q	69Q	69Q	69Q	73Q	76Q
EM100Q to EM5Q		80Q	77Q	77Q	71Q	69Q	70Q	69Q	72Q
0Q to 95Q					77Q	75Q	75Q	69Q	69Q
100Q to 195Q						81Q	81Q	73Q	70Q
200Q to 295Q								80Q	75Q
300Q to 395Q									81Q

Quantile Range	Grade 3			Grade 4			Grade 5		
	BOY	**MOY**	**EOY**	**BOY**	**MOY**	**EOY**	**BOY**	**MOY**	**EOY**
EM200Q to EM105Q	77Q								
EM100Q to EM5Q	72Q	77Q							
0Q to 95Q	69Q	72Q	76Q	75Q					
100Q to 195Q	70Q	69Q	71Q	71Q	76Q		78Q		
200Q to 295Q	74Q	70Q	69Q	69Q	71Q	75Q	74Q	76Q	78Q
300Q to 395Q	81Q	75Q	72Q	71Q	71Q	71Q	71Q	71Q	74Q
400Q to 495Q		80Q	78Q	77Q	73Q	70Q	71Q	70Q	70Q
500Q to 595Q					79Q	74Q	76Q	72Q	72Q
600Q to 695Q						79Q	82Q	78Q	76Q
700Q to 795Q									82Q

Quantile Range	Grade 6			Grade 7			Grade 8		
	BOY	**MOY**	**EOY**	**BOY**	**MOY**	**EOY**	**BOY**	**MOY**	**EOY**
200Q to 295Q	78Q								
300Q to 395Q	74Q	77Q							
400Q to 495Q	71Q	72Q	76Q	75Q	76Q	77Q	77Q		
500Q to 595Q	71Q	68Q	71Q	70Q	71Q	73Q	73Q	76Q	77Q
600Q to 695Q	75Q	69Q	70Q	69Q	69Q	69Q	69Q	71Q	72Q
700Q to 795Q	82Q	74Q	73Q	72Q	70Q	70Q	70Q	69Q	69Q
800Q to 895Q		79Q	79Q	78Q	75Q	75Q	75Q	72Q	70Q
900Q to 995Q						81Q	81Q	78Q	75Q
1000Q to 1095Q									81Q

Quantile Range	Grade 9			Grade 10			Grade 11/12		
	BOY	MOY	EOY	BOY	MOY	EOY	BOY	MOY	EOY
600Q to 695Q	75Q	75Q	76Q	76Q	78Q				
700Q to 795Q	71Q	71Q	71Q	71Q	74Q	76Q	75Q	76Q	78Q
800Q to 895Q	70Q	69Q	69Q	69Q	70Q	71Q	70Q	71Q	73Q
900Q to 995Q	73Q	73Q	71Q	71Q	71Q	69Q	69Q	69Q	69Q
1000Q to 1095Q	79Q	79Q	75Q	77Q	75Q	71Q	71Q	72Q	70Q
1100Q to 1195Q			81Q		81Q	77Q	77Q	78Q	75Q
1200Q to 1295Q									80Q

QSC Quantile Measure – Measurement Error

In a study of reading items, Stenner, Burdick, Sanford, and Burdick (2006) defined an ensemble to consist of all of the items that could be developed from a selected piece of text. This hierarchical theory (items and their use nested within the passage) is based on the notion of an ensemble as described by Einstein (1902) and Gibbs (1902). Stenner and his colleagues investigated the ensemble differences across items and it was determined that the Lexile measure of a piece of text is equivalent to the mean difficulty of the items associated with the passage.

The Quantile Framework is an extension of this ensemble theory and defines the ensemble to consist of all of the items that could be developed for a selected QSC at an introductory level. Each item that could be developed for a QSC will have a slightly different level of difficulty from other items developed for the same QSC when tested with students. These differences in difficulty can be due to such things as the wording in the stem, the level of the foils, how diagnostic the foils are, the extent of graphics utilized in the item, etc. The Quantile measure of an item within the LevelSet Math assessments is the mean difficulty level of the QSC ensemble.

Error may also be introduced when a QSC included at a certain grade level is not covered, or not covered at the same grade level, in a particular state curriculum. Although the grade level objectives and expectations are very similar across state curriculums, there are a handful of discrepancies that result in the same QSC being introduced at different grade levels. For example, basic division facts are introduced in Grade 3 in some states while other state curriculums consider it a Grade 4 topic.

Repeated observations of what is intended to be the "same thing" results in a series of non-identical numbers. Each measure resulting from the repetition of a measurement procedure is assumed to be exchangeable with any other measures that might have been made. In this approach there is not an *a priori* reason for favoring a measure obtained on Monday versus Tuesday or one based on multiple-choice versus constructed-response items. Any measure from this class of measures is specified to be equivalent. Stating that any measure from a defined class of measures is interchangeable does not imply that any measure at all will do—only any measure from the defined class. The mean of a sample of measures approaches a limit as the number of measures increases. The "closeness together" of the measures, expressed as a standard deviation, is an index of uncertainty regarding the magnitude of the quantity.

Observations that result from a theory-referenced measurement model are "generally objective," i.e., absolute measures are independent of the instrument used. Thus, a reader's 40-item response can be treated as a personal item bank. Or, a QSCs 6-item ensemble can be treated as a QSC item bank. One thousand "replicates" of the 40 items or the 6 items each can be sampled with replacement from this response record. Each replicate response is converted to a measure.

Various methods for obtaining variance estimates have been proposed in the literature. Three commonly used methods are the linearization (or Taylor) method, the jackknife method, and the balanced repeated replicates (BRR). Each of these methods has some limitations (Sitter, 1992). The linearization method requires theoretical calculation and programming of derivatives. The jackknife method, when the primary sampling units are selected without replacement, has only been developed for stratified sampling. The BRR method is only applicable to stratified sampling with restrictions on the within stratum sample sizes. To overcome some of these limitations, various bootstrap methods have been suggested. "Bootstrap methods reutilize the existing estimation system repeatedly, using computer power to avoid theoretical work" (p. 136).

To compute standard error of measurements, resampling needs to be done as near the original sampling frame as possible. The only thing that changes from one form of a test to another are the QSCs selected and the individual items within the QSC. Therefore, the re-sampling frame consists of reselecting QSCs and items and calculating the variance related to the use of different QSCs and items in a test form. To obtain the bootstrap estimate of a parameter, the mean of each set of *n* estimates sampled with replacement is calculated (*n* is typically 1,000), and the standard error of each of the statistics is the standard deviation computed from the sampling distribution. The bootstrap parameter estimates are not used; only the standard deviations are used to estimate the variability that would be expected if the study had been conducted 1,000 times.

A study was conducted in May 2014 with a large bank of items calibrated to the Quantile scale. The bank consisted of 5,291 items across 415 QSCs of the Quantile Framework. A random selection of 30 QSCs was selected without replacement and then a random selection of 6 items from each QSC was selected without replacement. The process was then repeated 1,000 times. The resulting theory misspecification error of a QSC was estimated to be 68.73Q. Thus, for a QSC with a Quantile measure of 670Q, the expectation is that 90% of such items developed for the QSC will have "true" item difficulties within an interval of +/-137Q of the QSC Quantile measure.

Validity

The 2014 *Standards for Educational and Psychological Testing* (American Educational Research Association, American Psychological Association, and National Council on Measurement in Education) state that "validity refers to the degree to which evidence and theory support the interpretations of test scores for proposed uses of tests" (p. 11). Validity evidence provides information about how well a test will fulfill its intended function. "The process of ascribing meaning to scores produced by a measurement procedure is generally recognized as the most important task in developing an educational or psychological measure, be it an achievement test, interest inventory, or personality scale" (Stenner, Smith, and Burdick, 1983).

Because a score from the LevelSet Math assessments will be used as a measure of the mathematical ability of a student and will be used to target mathematical instruction, validity evidence should primarily focus on the degree to which the LevelSet Math assessments measure mathematical achievement and demonstrates understanding of grade-appropriate mathematical skills. For convenience, two sources of validity evidence—content and construct validity evidence—will be described as if they are unique, independent components rather than interrelated parts. At this time the primary source of validity evidence comes from an examination of the content of the LevelSet Math assessments (content validity) and the degree to which the assessment can be said to measure mathematical achievement (construct validity evidence). As more data are collected and more studies are completed, additional validity evidence will be described.

Content Validity Evidence

Validity evidence for the content of a test relates to the degree to which the test content is supportive of the intended interpretations of the test scores. The content validity of a test refers to the adequacy with which relevant content has been sampled and represented in the test. Content validity was also built into the LevelSet Math assessments during its development. The LevelSet Math assessments were designed to measure readiness for mathematical instruction. To this end, the test forms were constructed with content skills in mind. All items were written and reviewed by experienced classroom teachers to ensure that the content of the items was developmentally appropriate and representative of classroom experiences and that the distractors were reflective of common student errors.

The content validity of the LevelSet Math assessments is based on the alignment between the content of the items and the curricular framework used to develop the assessment. Each item on the assessment was aligned with a specific QSC in the Quantile Framework. In addition, each QSC has been aligned with a specific standard in the Common Core State Standards for Mathematics (CCSS) curriculum standards.

The development of the CCSS standards has established a clear set of K-12 standards that will enable all students to become increasingly more proficient in understanding and utilizing mathematics—with steady advancement to college and career readiness by high school graduation. To see the alignment between the Quantile Framework QSCs and the CCSS standards, visit https://hub.lexile.com/math-skills-database and search standards for "Common Core."

The CCSS mathematics standards stress both procedural skill and conceptual understanding to prepare students for the challenges of their secondary and postsecondary pursuits. They lay the groundwork for K-5 students to learn about whole numbers, operations, fractions, and decimals, all of which are required to learn more challenging concepts and procedures.

The mathematics standards outline practices that each student should develop in the early grades and then master as they progress through middle and high school:

1. Make sense of problems and persevere in solving by formulating plans, strategizing, and evaluating the process.
2. Apply mathematics to everyday life problems in society.
3. Analyze mathematical relationships and communicate mathematical ideas.
4. Use appropriate tools strategically.
5. Attend to precision.
6. Look for and make use of structure.
7. Look for and express regularity in repeated reasoning.

For more information on the content validity of the LevelSet Math assessments and the Quantile Framework, please refer to the other sections of this guide (sections entitled "The Quantile Framework for Mathematics," "The Quantile Item Bank," and "Development of the LevelSet Math assessments").

Construct Validity Evidence

Evidence for the construct validity of the LevelSet Math assessments is provided by the body of research supporting The Quantile Framework for Mathematics. The development of the LevelSet Math assessments utilized the Quantile Framework and the calibration of items specified to previously field-tested and analyzed items. Item writers for the Quantile Item Bank were provided training on item development that matched the training used during the development of the Quantile item bank, and item reviewers had access to all items from the Quantile item bank. These items had been previously calibrated to the Quantile scale to ensure that items developed for the Quantile Item Bank were theoretically consistent with other items calibrated to the Quantile scale, and that they maintained their individual item calibrations.

Prior research has shown that test scores derived from items calibrated from the Quantile field study are highly correlated with other assessments of mathematics achievement. The section in this technical report entitled "The Quantile Framework for Mathematics" provides a detailed description of the framework and its construct validity. The section also includes evidence to support the fact that tests based upon the framework can accurately measure mathematics achievement.

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Appendices

Appendix A

The Quantile® Framework for Mathematics Map

THE QUANTILE® FRAMEWORK FOR MATHEMATICS MAP

Linking assessment with mathematics instruction

Imagine empowering and accelerating students' learning in mathematics by better differentiating instruction and monitoring growth in student ability. With the Quantile Framework, educators can help achieve this goal by identifying level-appropriate mathematical tasks for students and track their progress!

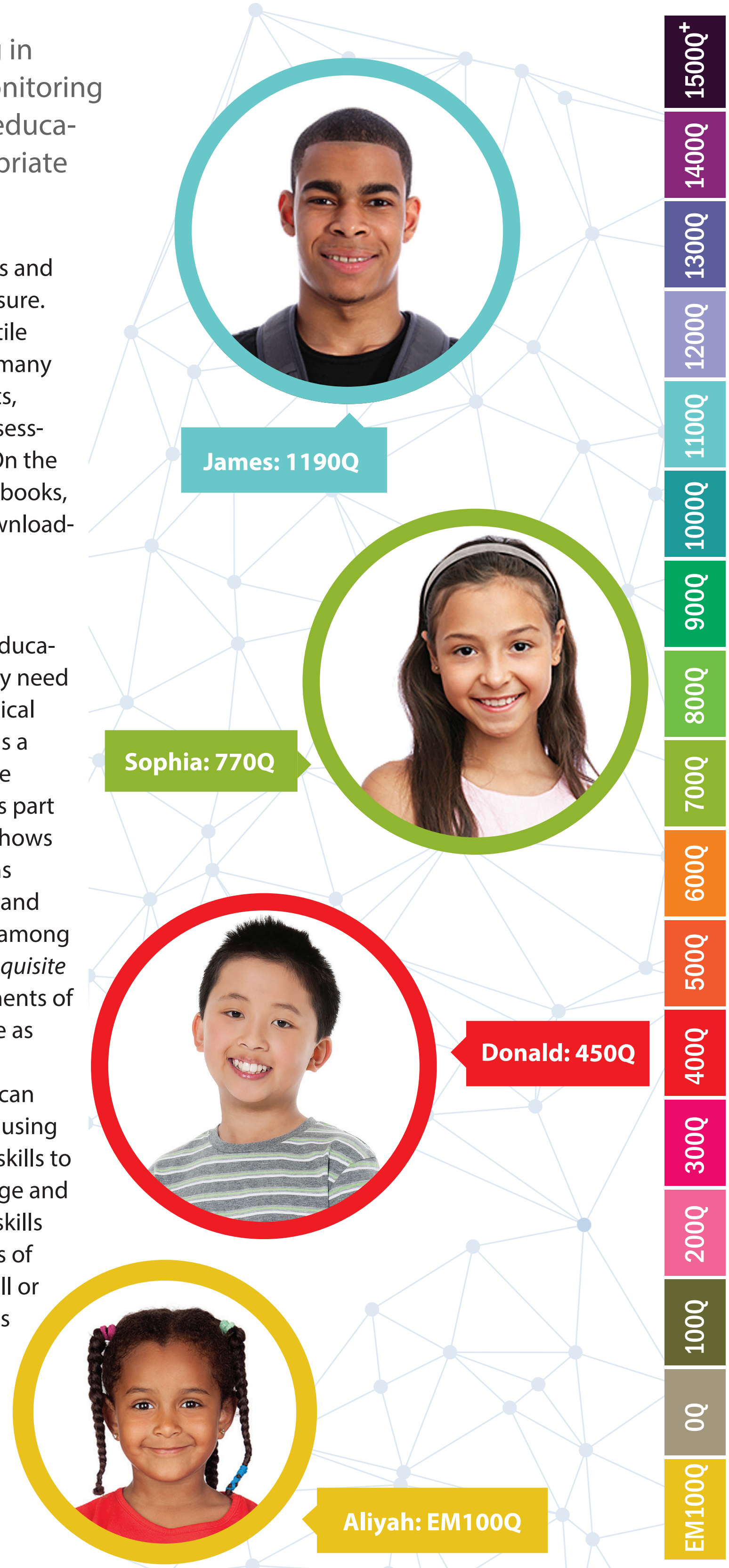
HOW IT WORKS

The Quantile Framework for Mathematics is a unique measurement system that uses a common scale and metric to assess a student's mathematical achievement level and the difficulty of specific skills and concepts. The Quantile Framework describes a student's ability to solve mathematical problems and the demand of the skills and concepts typically taught in kindergarten mathematics through Algebra II, Geometry, Trigonometry and Precalculus. The Quantile Map provides educators with a sampling of primary mathematical skills and concepts from over 500 Quantile Skills and Concepts (QSCs) throughout the Quantile scale. This sampling of QSCs ranges from EM (Emerging Mathematician) for early, foundational mathematical skills and concepts to 1500Q for more advanced skills and concepts. As the difficulty, or demand of the skill increases, so does the Quantile measure.

HOW TO USE IT

With the Quantile Framework, educators can explore the interconnectedness of mathematical skills and concepts and identify those elements that are critical for progressing student learning. Educators are better able to inform their instruction on how to best teach a skill or concept by pinpointing which skills build upon each other. The skill mapping of mathematical concepts enables educators to build an instructional path that best fits their students' unique abilites. Both students and QSCs receive a Quantile measure. Numerous tests report Quantile student measures including many state end-of-year assessments, national norm-referenced assessments and math programs. On the QSC side, more than 580 textbooks, 64,000 lessons and 3,100 downloadable resources have received Quantile measures.

Quantile measures provide educators with the information they need to identify gaps in mathematical knowledge, as well as serve as a guide for progressing to more advanced topics. Every QSC is part of a knowledge cluster that shows relationships and connections between mathematical skills and offers their relative difficulty among different skills. Both the *prerequisite* and *impending* skills are elements of knowledge clusters and serve as building blocks that support students' success. Educators can advance student learning by using prerequisite and impending skills to build mathematical knowledge and understanding. Prerequisite skills help educators see the pieces of the puzzle that make up a skill or concept, showing what needs to be understood first. Impending skills are skills and concepts that build upon a focus skill and allow educators to see a trajectory of knowledge across grades and content strands.



For more information, free resources, and to search the Math Skills Database, visit Quantiles.com.

THE QUANTILE® FRAMEWORK FOR MATHEMATICS MAP

Linking assessment with mathematics instruction

High School Example
James

Heritage High School | Geometry Course

Quantile Measure: 1190Q

James is exploring theorems about lines and angles in his Geometry class. In his current learning path, the focus skill being taught is *use properties, definitions, and theorems of angles and lines to solve problems related to adjacent, vertical, complementary, supplementary, and linear pairs of angles.* This focus skill is part of a knowledge cluster that contains prerequisite and impending skills. Working with prerequisite skills can help students struggling to learn and impending skills can help students progress to the next level of learning.

Since James' Quantile measure is within the range of the focus skill being taught (his Quantile measure +/- 50Q), James will be ready for this type of instruction. With his mathematical ability being at the same level as the focus skill, learning will be optimal. Once James is performing well with the focus skill, he will be better prepared to learn the impending skills connected with this focus skill.

1250Q
IMPENDING SKILL
Use definitions and theorems of angles formed when a transversal intersects parallel lines.

1220Q
IMPENDING SKILL
Use properties, definitions, and theorems of polygons to solve problems related to the interior and exterior angles of a convex polygon.

1160Q
FOCUS SKILL
Use properties, definitions, and theorems of angles and lines to solve problems related to adjacent, vertical, complementary, supplementary, and linear pairs of angles.
CCSS G.CO.9

1010Q
PREREQUISITE SKILL
Define and identify alternate interior, alternate exterior, corresponding, adjacent and vertical angles.

1020Q
PREREQUISITE SKILL
Define and identify complementary and supplementary angles.

800Q
PREREQUISITE SKILL
Write a linear equation or inequality to represent a given number or word problem; solve.

ALGEBRA & ALGEBRAIC THINKING | NUMBER SENSE | NUMERICAL OPERATIONS | MEASUREMENT | GEOMETRY | DATA ANALYSIS, STATISTICS & PROBABILITY

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THE **QUANTILE® FRAMEWORK FOR MATHEMATICS** MAP

Linking assessment with mathematics instruction

Middle School Example
Sophia

Heritage Middle School | Grade 6

Quantile Measure: 770Q

Sophia is using variables to represent mathematical expressions in her math class. In her current learning path, the focus skill being taught is *translate between models or verbal phrases and algebraic expressions*. This focus skill is part of a knowledge cluster that contains prerequisite and impending skills. Working with prerequisite skills can help students struggling to learn and impending skills can help students progress to the next level of learning.

Since Sophia's Quantile measure is within the range of the focus skill being taught (her Quantile measure +/- 50Q), Sophia will be ready for this type of instruction. With her mathematical ability being at the same level as the focus skill, learning will be optimal. Once Sophia is performing well with the focus skill, she will be better prepared to learn the impending skills connected with this focus skill.

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810Q ◆

IMPENDING SKILL

Write an equation to describe the algebraic relationship between two defined variables in number and word problems, including recognizing which variable is dependent.

◆ **800Q**

IMPENDING SKILL

Identify parts of a numerical or algebraic expression.

◆ **800Q**

IMPENDING SKILL

Write a linear equation or inequality to represent a given number or word problem; solve.

750Q ◆

FOCUS SKILL

Translate between models or verbal phrases and algebraic expressions.

CCSS 6.EE.6

◆ **620Q**

PREREQUISITE SKILL

Translate between models or verbal phrases and numerical expressions.

◆ **430Q**

PREREQUISITE SKILL

Describe the meaning of an unknown in the context of a word problem.

◆ ALGEBRA & ALGEBRAIC THINKING | ★ NUMBER SENSE | ■ NUMERICAL OPERATIONS | ● MEASUREMENT | ⏢ GEOMETRY | ▲ DATA ANALYSIS, STATISTICS & PROBABILITY

THE QUANTILE® FRAMEWORK FOR MATHEMATICS MAP

Linking assessment with mathematics instruction

Late Elementary Example Donald

Heritage Elementary School | Grade 4

Student Quantile Measure: 450Q

Donald is learning about line graphs with very large data values. In his current learning path, the focus skill being taught is *organize, display, and interpret information in graphs containing scales that represent multiple units.* This focus skill is part of a knowledge cluster that contains prerequisite and impending skills. Working with prerequisite skills can help students struggling to learn and impending skills can help students progress to the next level of learning.

Since Donald's Quantile measure is within the range of the focus skill being taught (his Quantile measure +/- 50Q), Donald will be ready for this type of instruction. With his mathematical ability being at the same level as the focus skill, learning will be optimal. Once Donald is performing well with the focus skill, he will be better prepared to learn the impending skills connected with this focus skill.

800Q
IMPENDING SKILL
Identify and use appropriate scales and intervals in graphs and data displays.

480Q
FOCUS SKILL
Organize, display, and interpret information in graphs containing scales that represent multiple units.
CCSS 3.MD.3

480Q
IMPENDING SKILL
Organize, display, and interpret information in bar graphs.

470Q
IMPENDING SKILL
Organize, display, and interpret information in line graphs.

200Q
PREREQUISITE SKILL
Organize, display, and interpret information in line plots and tally charts.

110Q
PREREQUISITE SKILL
Skip count by 2s, 5s and 10s beginning at any number.

90Q
PREREQUISITE SKILL
Skip count by 3s, 4s, 6s, 7s, 8s, and 9s.

EM10Q
PREREQUISITE SKILL
Organize, display, and interpret information in picture graphs and bar graphs using grids.

MetaMetrics

For more information, visit Quantiles.com.

ALGEBRA & ALGEBRAIC THINKING | NUMBER SENSE | NUMERICAL OPERATIONS | MEASUREMENT | GEOMETRY | DATA ANALYSIS , STATISTICS & PROBABILITY

THE **QUANTILE® FRAMEWORK FOR MATHEMATICS** MAP

Linking assessment with mathematics instruction

Early Elementary Example
Aliyah

Heritage Elementary School | Kindergarten

Quantile Measure: EM100Q

Aliyah is exploring unknown-addend problems in her class. In her current learning path, the focus skill being taught is *know and use related addition and subtraction facts.* This focus skill is part of a knowledge cluster that contains prerequisite and impending skills. Working with prerequisite skills can help students struggling to learn and impending skills can help students progress to the next level of learning.

Since Aliyah's Quantile measure is within the range of the focus skill being taught (her Quantile measure +/- 50Q), Aliyah will be ready for this type of instruction. With her mathematical ability being at the same level as the focus skill, learning will be optimal. Once Aliyah is performing well with the focus skill, she will be better prepared to learn the impending skills connected with this focus skill.

EM25Q ■
IMPENDING SKILL

Model the concept of subtraction using numbers less than or equal to 10.

EM80Q ■
FOCUS SKILL

Know and use related addition and subtraction facts.

CCSS 1.OA.4

EM110Q ◆
PREREQUISITE SKILL

Identify missing addends for addition facts.

■ EM260Q
PREREQUISITE SKILL

Model the concept of addition for sums to 10.

MetaMetrics.

For more information, visit Quantiles.com.

◆ ALGEBRA & ALGEBRAIC THINKING | ★ NUMBER SENSE | ■ NUMERICAL OPERATIONS | ● MEASUREMENT | ⏢ GEOMETRY | ▲ DATA ANALYSIS, STATISTICS & PROBABILITY

Grade K

Form	QSC	Description
BOY	2	Read, write, and count using whole numbers; rote count to 30.
	4	Read and write numerals using one-to-one correspondence to match sets of 0 to 10.
	6	Use ordinal numbers first through tenth to describe order.
	7	Create and identify sets with greater than, less than, or equal number of members by matching.
	14	Describe, compare, and order objects using mathematical vocabulary.
	15	Use directional and positional words.
	16	Describe likenesses and differences between and among objects.
	24	Rote count beginning at 1 or at another number by 1s, and rote count by 2s, 5s and 10s to 100 beginning at 2, 5, or 10.
	25	Read and write numerals using one-to-one correspondence to match sets of 11 to 100.
	33	Represent numbers up to 100 in a variety of ways such as tallies, ten frames, and other models.
	36	Model the concept of addition for sums to 10.
	39	Recognize the context in which addition or subtraction is appropriate, and write number sentences to solve number or word problems.
	41	Know and use addition and subtraction facts to 10 and understand the meaning of equality.
	536	Identify, draw, and name basic shapes such as triangles, squares, rectangles, hexagons, and circles.
	542	Combine two- and three- dimensional simple figures to create a composite figure.
	663	Represent a number in a variety of numerical ways.
MOY	2	Read, write, and count using whole numbers; rote count to 30.
	4	Read and write numerals using one-to-one correspondence to match sets of 0 to 10.
	6	Use ordinal numbers first through tenth to describe order.
	7	Create and identify sets with greater than, less than, or equal number of members by matching.
	15	Use directional and positional words.
	16	Describe likenesses and differences between and among objects.
	24	Rote count beginning at 1 or at another number by 1s, and rote count by 2s, 5s and 10s to 100 beginning at 2, 5, or 10.
	25	Read and write numerals using one-to-one correspondence to match sets of 11 to 100.
	33	Represent numbers up to 100 in a variety of ways such as tallies, ten frames, and other models.
	36	Model the concept of addition for sums to 10.
	37	Model the concept of subtraction using numbers less than or equal to 10.

Form	QSC	Description
	39	Recognize the context in which addition or subtraction is appropriate, and write number sentences to solve number or word problems.
	41	Know and use addition and subtraction facts to 10 and understand the meaning of equality.
	46	Identify and name basic solid figures: rectangular prism, cylinder, pyramid, and cone; identify in the environment.
	54	Sort a set of objects in one or more ways; explain.
	581	Measure length using nonstandard units.
	663	Represent a number in a variety of numerical ways.
	1001	Compare and order sets and numerals up to 20, including using symbol notation (>, <, =).
EOY	2	Read, write, and count using whole numbers; rote count to 30.
	4	Read and write numerals using one-to-one correspondence to match sets of 0 to 10.
	7	Create and identify sets with greater than, less than, or equal number of members by matching.
	14	Describe, compare, and order objects using mathematical vocabulary.
	16	Describe likenesses and differences between and among objects.
	24	Rote count beginning at 1 or at another number by 1s, and rote count by 2s, 5s and 10s to 100 beginning at 2, 5, or 10.
	25	Read and write numerals using one-to-one correspondence to match sets of 11 to 100.
	33	Represent numbers up to 100 in a variety of ways such as tallies, ten frames, and other models.
	35	Use place value with ones and tens.
	36	Model the concept of addition for sums to 10.
	39	Recognize the context in which addition or subtraction is appropriate, and write number sentences to solve number or word problems.
	41	Know and use addition and subtraction facts to 10 and understand the meaning of equality.
	46	Identify and name basic solid figures: rectangular prism, cylinder, pyramid, and cone; identify in the environment.
	54	Sort a set of objects in one or more ways; explain.
	582	Measure weight using nonstandard units.
	663	Represent a number in a variety of numerical ways.
	1001	Compare and order sets and numerals up to 20, including using symbol notation (>, <, =).

Grade 1

Form	QSC	Description
BOY	2	Read, write, and count using whole numbers; rote count to 30.
	4	Read and write numerals using one-to-one correspondence to match sets of 0 to 10.
	6	Use ordinal numbers first through tenth to describe order.
	7	Create and identify sets with greater than, less than, or equal number of members by matching.
	16	Describe likenesses and differences between and among objects.
	24	Rote count beginning at 1 or at another number by 1s, and rote count by 2s, 5s and 10s to 100 beginning at 2, 5, or 10.
	35	Use place value with ones and tens.
	37	Model the concept of subtraction using numbers less than or equal to 10.
	39	Recognize the context in which addition or subtraction is appropriate, and write number sentences to solve number or word problems.
	41	Know and use addition and subtraction facts to 10 and understand the meaning of equality.
	46	Identify and name basic solid figures: rectangular prism, cylinder, pyramid, and cone; identify in the environment.
	54	Sort a set of objects in one or more ways; explain.
	581	Measure length using nonstandard units.
	582	Measure weight using nonstandard units.
	663	Represent a number in a variety of numerical ways.
	1001	Compare and order sets and numerals up to 20, including using symbol notation (>, <, =).
MOY	14	Describe, compare, and order objects using mathematical vocabulary.
	24	Rote count beginning at 1 or at another number by 1s, and rote count by 2s, 5s and 10s to 100 beginning at 2, 5, or 10.
	26	Compare and order sets and numerals from 21 to 100, including using symbol notation (>, <, =).
	35	Use place value with ones and tens.
	38	Model the division of sets or the partition of figures into two, three or four equal parts (fair shares).
	46	Identify and name basic solid figures: rectangular prism, cylinder, pyramid, and cone; identify in the environment.
	79	Add 2- and 3-digit numbers with and without models for number and word problems that do not require regrouping.
	161	Use the commutative and associative properties to add or multiply numerical expressions.
	544	Write an addition or a subtraction sentence that represents a number or word problem; solve.
	581	Measure length using nonstandard units.
	583	Measure capacity using nonstandard units.
	599	Subtract 2- and 3-digit numbers with and without models for number and word problems that do not require regrouping.
	663	Represent a number in a variety of numerical ways.

Form	QSC	Description
	1001	Compare and order sets and numerals up to 20, including using symbol notation (>, <, =).
	1004	Find the unknown in an addition or subtraction number sentence.
	1005	Tell time to the nearest hour and half-hour using digital and analog clocks.
EOY	26	Compare and order sets and numerals from 21 to 100, including using symbol notation (>, <, =).
	35	Use place value with ones and tens.
	38	Model the division of sets or the partition of figures into two, three or four equal parts (fair shares).
	59	Answer comparative and quantitative questions about charts and graphs.
	60	Organize, display, and interpret information in line plots and tally charts.
	161	Use the commutative and associative properties to add or multiply numerical expressions.
	544	Write an addition or a subtraction sentence that represents a number or word problem; solve.
	581	Measure length using nonstandard units.
	598	Add 2- and 3-digit numbers with and without models for number and word problems that require regrouping.
	599	Subtract 2- and 3-digit numbers with and without models for number and word problems that do not require regrouping.
	1002	Use models and appropriate vocabulary to determine properties of basic plane figures (open or closed, number of sides and vertices or corners).
	1004	Find the unknown in an addition or subtraction number sentence.
	1005	Tell time to the nearest hour and half-hour using digital and analog clocks.

Grade 2

Form	QSC	Description
BOY	26	Compare and order sets and numerals from 21 to 100, including using symbol notation (>, <, =).
	35	Use place value with ones and tens.
	60	Organize, display, and interpret information in line plots and tally charts.
	119	Use the identity properties for addition and multiplication and the zero property for multiplication.
	161	Use the commutative and associative properties to add or multiply numerical expressions.
	581	Measure length using nonstandard units.
	598	Add 2- and 3-digit numbers with and without models for number and word problems that require regrouping.
	599	Subtract 2- and 3-digit numbers with and without models for number and word problems that do not require regrouping.
	663	Represent a number in a variety of numerical ways.
	1002	Use models and appropriate vocabulary to determine properties of basic plane figures (open or closed, number of sides and vertices or corners).
	1004	Find the unknown in an addition or subtraction number sentence.
	1005	Tell time to the nearest hour and half-hour using digital and analog clocks.
MOY	38	Model the division of sets or the partition of figures into two, three or four equal parts (fair shares).
	60	Organize, display, and interpret information in line plots and tally charts.
	71	Use place value with hundreds.
	73	Indicate the value of each digit in any 2- or 3-digit number.
	97	Locate points on a number line.
	110	Relate standard and expanded notation to 3- and 4-digit numbers.
	111	Compare and order numbers less than 10,000.
	113	Identify odd and even numbers.
	117	Subtract 2- and 3-digit numbers with and without models for number and word problems that require regrouping.
	118	Model multiplication in a variety of ways including grouping objects, repeated addition, rectangular arrays, skip counting, and area models.
	161	Use the commutative and associative properties to add or multiply numerical expressions.
	174	Use manipulatives, pictorial representations, and appropriate vocabulary (e.g., polygon, side, angle, vertex, diameter) to identify and compare properties of plane figures.
	540	Identify combinations of fractions that make one whole.
	541	Tell time at the five-minute intervals.
	544	Write an addition or a subtraction sentence that represents a number or word problem; solve.
	598	Add 2- and 3-digit numbers with and without models for number and word problems that require regrouping.

Form	QSC	Description
	599	Subtract 2- and 3-digit numbers with and without models for number and word problems that do not require regrouping.
	1005	Tell time to the nearest hour and half-hour using digital and analog clocks.
	1006	Write an addition and subtraction sentence that represents a two-step word problem; solve.
EOY	60	Organize, display, and interpret information in line plots and tally charts.
	73	Indicate the value of each digit in any 2- or 3-digit number.
	97	Locate points on a number line.
	106	Make different sets of coins with equivalent values.
	111	Compare and order numbers less than 10,000.
	117	Subtract 2- and 3-digit numbers with and without models for number and word problems that require regrouping.
	118	Model multiplication in a variety of ways including grouping objects, repeated addition, rectangular arrays, skip counting, and area models.
	147	Determine the value of sets of coins and bills using cent sign and dollar sign appropriately. Create equivalent amounts with different coins and bills.
	148	Estimate and compute the cost of items greater than $1.00; make change.
	161	Use the commutative and associative properties to add or multiply numerical expressions.
	174	Use manipulatives, pictorial representations, and appropriate vocabulary (e.g., polygon, side, angle, vertex, diameter) to identify and compare properties of plane figures.
	175	Use manipulatives, pictorial representations, and appropriate vocabulary (e.g., face, edge, vertex, and base) to identify and compare properties of solid figures.
	540	Identify combinations of fractions that make one whole.
	541	Tell time at the five-minute intervals.
	544	Write an addition or a subtraction sentence that represents a number or word problem; solve.
	649	Estimate, measure, and compare length using appropriate tools and units in number and word problems.
	1006	Write an addition and subtraction sentence that represents a two-step word problem; solve.
	1007	Skip count by 2s, 5s and 10s beginning at any number.

Grade 3

Form	QSC	Description
BOY	60	Organize, display, and interpret information in line plots and tally charts.
	73	Indicate the value of each digit in any 2- or 3-digit number.
	97	Locate points on a number line.
	106	Make different sets of coins with equivalent values.
	111	Compare and order numbers less than 10,000.
	117	Subtract 2- and 3-digit numbers with and without models for number and word problems that require regrouping.
	118	Model multiplication in a variety of ways including grouping objects, repeated addition, rectangular arrays, skip counting, and area models.
	119	Use the identity properties for addition and multiplication and the zero property for multiplication.
	147	Determine the value of sets of coins and bills using cent sign and dollar sign appropriately. Create equivalent amounts with different coins and bills.
	148	Estimate and compute the cost of items greater than $1.00; make change.
	174	Use manipulatives, pictorial representations, and appropriate vocabulary (e.g., polygon, side, angle, vertex, diameter) to identify and compare properties of plane figures.
	175	Use manipulatives, pictorial representations, and appropriate vocabulary (e.g., face, edge, vertex, and base) to identify and compare properties of solid figures.
	541	Tell time at the five-minute intervals.
	544	Write an addition or a subtraction sentence that represents a number or word problem; solve.
	1006	Write an addition and subtraction sentence that represents a two-step word problem; solve.
	1007	Skip count by 2s, 5s and 10s beginning at any number.
MOY	83	Identify, draw, and name shapes such as quadrilaterals, trapezoids, parallelograms, rhombi, and pentagons.
	116	Use models to write equivalent fractions, including using composition or decomposition or showing relationships among halves, fourths, and eighths, and thirds and sixths.
	117	Subtract 2- and 3-digit numbers with and without models for number and word problems that require regrouping.
	118	Model multiplication in a variety of ways including grouping objects, repeated addition, rectangular arrays, skip counting, and area models.
	120	Model division in a variety of ways including sharing equally, repeated subtraction, rectangular arrays, and the relationship with multiplication.
	121	Use multiplication facts through 144.
	129	Describe and demonstrate patterns in skip counting and multiplication; continue sequences beyond memorized or modeled numbers.
	134	Organize, display, and interpret information in bar graphs.

Form	QSC	Description
	147	Determine the value of sets of coins and bills using cent sign and dollar sign appropriately. Create equivalent amounts with different coins and bills.
	148	Estimate and compute the cost of items greater than $1.00; make change.
	162	Know and use division facts related to multiplication facts through 144.
	174	Use manipulatives, pictorial representations, and appropriate vocabulary (e.g., polygon, side, angle, vertex, diameter) to identify and compare properties of plane figures.
	538	Compare fractions with the same numerator or denominator concretely and symbolically.
	540	Identify combinations of fractions that make one whole.
	578	Use the distributive property to represent and simplify numerical expressions.
	607	Write a multiplication or a division sentence that represents a number or word problem; solve.
	660	Round whole numbers to a given place value.
	1008	Write number sentences using any combination of the four operations that represent a two-step word problem; solve.
	1010	Multiply a 1-digit number by a 2-digit multiple of 10.
EOY	116	Use models to write equivalent fractions, including using composition or decomposition or showing relationships among halves, fourths, and eighths, and thirds and sixths.
	117	Subtract 2- and 3-digit numbers with and without models for number and word problems that require regrouping.
	120	Model division in a variety of ways including sharing equally, repeated subtraction, rectangular arrays, and the relationship with multiplication.
	129	Describe and demonstrate patterns in skip counting and multiplication; continue sequences beyond memorized or modeled numbers.
	134	Organize, display, and interpret information in bar graphs.
	146	Determine perimeter using concrete models, nonstandard units, and standard units in number and word problems.
	162	Know and use division facts related to multiplication facts through 144.
	174	Use manipulatives, pictorial representations, and appropriate vocabulary (e.g., polygon, side, angle, vertex, diameter) to identify and compare properties of plane figures.
	191	Use grids to develop the relationship between the total numbers of square units in a rectangle and the length and width of the rectangle (l x w); find area using the formula in number and word problems.
	538	Compare fractions with the same numerator or denominator concretely and symbolically.
	578	Use the distributive property to represent and simplify numerical expressions.
	607	Write a multiplication or a division sentence that represents a number or word problem; solve.

Form	QSC	Description
	620	Name polygons by the number of sides. Distinguish quadrilaterals based on properties of their sides or angles; relate quadrilaterals based upon their hierarchical attributes.
	660	Round whole numbers to a given place value.
	1008	Write number sentences using any combination of the four operations that represent a two-step word problem; solve.
	1009	Apply appropriate types of estimation for number and word problems that include estimating products and quotients.
	1010	Multiply a 1-digit number by a 2-digit multiple of 10.
	1012	Organize, display, and interpret information in line plots with a horizontal scale in fractional units.
	1018	Use models to develop the relationship between the total distance around a figure and the formula for perimeter; find perimeter using the formula in number and word problems.

Grade 4

Form	QSC	Description
BOY	115	Use benchmark numbers (zero, one-half, one) and models to compare and order fractions.
	116	Use models to write equivalent fractions, including using composition or decomposition or showing relationships among halves, fourths, and eighths, and thirds and sixths.
	118	Model multiplication in a variety of ways including grouping objects, repeated addition, rectangular arrays, skip counting, and area models.
	119	Use the identity properties for addition and multiplication and the zero property for multiplication.
	120	Model division in a variety of ways including sharing equally, repeated subtraction, rectangular arrays, and the relationship with multiplication.
	134	Organize, display, and interpret information in bar graphs.
	146	Determine perimeter using concrete models, nonstandard units, and standard units in number and word problems.
	153	Apply appropriate type of estimation for sums and differences.
	162	Know and use division facts related to multiplication facts through 144.
	191	Use grids to develop the relationship between the total numbers of square units in a rectangle and the length and width of the rectangle (l x w); find area using the formula in number and word problems.
	192	Determine the area of rectangles, squares, and composite figures using nonstandard units, grids, and standard units in number and word problems.
	538	Compare fractions with the same numerator or denominator concretely and symbolically.
	578	Use the distributive property to represent and simplify numerical expressions.
	596	Solve problems involving elapsed time.
	607	Write a multiplication or a division sentence that represents a number or word problem; solve.
	620	Name polygons by the number of sides. Distinguish quadrilaterals based on properties of their sides or angles; relate quadrilaterals based upon their hierarchical attributes.
	660	Round whole numbers to a given place value.
	1008	Write number sentences using any combination of the four operations that represent a two-step word problem; solve.
	1009	Apply appropriate types of estimation for number and word problems that include estimating products and quotients.
	1010	Multiply a 1-digit number by a 2-digit multiple of 10.
	1012	Organize, display, and interpret information in line plots with a horizontal scale in fractional units.
MOY	115	Use benchmark numbers (zero, one-half, one) and models to compare and order fractions.
	136	Organize, display, and interpret information in graphs containing scales that represent multiple units.

Form	QSC	Description
	157	Rewrite and compare decimals to fractions (tenths and hundredths) with and without models and pictures.
	163	Understand that many whole numbers factor in different ways.
	165	Multiply a multi-digit whole number by a 1-digit whole number or a 2-digit multiple of 10.
	166	Divide using single-digit divisors with and without remainders.
	191	Use grids to develop the relationship between the total numbers of square units in a rectangle and the length and width of the rectangle (l x w); find area using the formula in number and word problems.
	199	Add and subtract fractions and mixed numbers with like denominators (without regrouping) in number and word problems.
	221	Find multiples, common multiples, and the least common multiple of numbers; explain.
	222	Find factors, common factors, and the greatest common factor of numbers; explain.
	223	Identify prime and composite numbers less than 100.
	231	Add and subtract fractions and mixed numbers with unlike denominators in number and word problems.
	578	Use the distributive property to represent and simplify numerical expressions.
	596	Solve problems involving elapsed time.
	607	Write a multiplication or a division sentence that represents a number or word problem; solve.
	620	Name polygons by the number of sides. Distinguish quadrilaterals based on properties of their sides or angles; relate quadrilaterals based upon their hierarchical attributes.
	1008	Write number sentences using any combination of the four operations that represent a two-step word problem; solve.
	1009	Apply appropriate types of estimation for number and word problems that include estimating products and quotients.
	1012	Organize, display, and interpret information in line plots with a horizontal scale in fractional units.
	1014	Indicate and compare the place value of each digit in a multi-digit whole number or decimal.
	1017	Use models to represent a fraction as a product of a whole number and a unit fraction in number and word problems.
	1018	Use models to develop the relationship between the total distance around a figure and the formula for perimeter; find perimeter using the formula in number and word problems.
EOY	155	Compare and order fractions using common numerators or denominators.
	166	Divide using single-digit divisors with and without remainders.
	173	Identify, draw, and name: points, rays, line segments, lines, and planes.
	180	Construct or complete a table of values to solve problems associated with a given relationship.
	221	Find multiples, common multiples, and the least common multiple of numbers; explain.
	222	Find factors, common factors, and the greatest common factor of numbers; explain.

Form	QSC	Description
	223	Identify prime and composite numbers less than 100.
	224	Multiply two fractions or a fraction and a whole number in number and word problems.
	231	Add and subtract fractions and mixed numbers with unlike denominators in number and word problems.
	258	Convert measures of length, area, capacity, weight, and time expressed in a given unit to other units in the same measurement system in number and word problems.
	266	Use remainders in problem-solving situations and interpret the remainder with respect to the original problem.
	607	Write a multiplication or a division sentence that represents a number or word problem; solve.
	620	Name polygons by the number of sides. Distinguish quadrilaterals based on properties of their sides or angles; relate quadrilaterals based upon their hierarchical attributes.
	624	Identify and classify triangles according to the measures of the interior angles and the lengths of the sides; relate triangles based upon their hierarchical attributes.
	660	Round whole numbers to a given place value.
	668	Write equivalent fractions with smaller or larger denominators.
	1012	Organize, display, and interpret information in line plots with a horizontal scale in fractional units.
	1014	Indicate and compare the place value of each digit in a multi-digit whole number or decimal.
	1015	Add multi-digit numbers with regrouping in number and word problems.
	1018	Use models to develop the relationship between the total distance around a figure and the formula for perimeter; find perimeter using the formula in number and word problems.
	1019	Use addition and subtraction to find unknown measures of non-overlapping angles.
	1023	Add and subtract fractions and mixed numbers using models and pictures to explain the process and record the results in number and word problems.

Grade 5

Form	QSC	Description
BOY	155	Compare and order fractions using common numerators or denominators.
	157	Rewrite and compare decimals to fractions (tenths and hundredths) with and without models and pictures.
	166	Divide using single-digit divisors with and without remainders.
	173	Identify, draw, and name: points, rays, line segments, lines, and planes.
	180	Construct or complete a table of values to solve problems associated with a given relationship.
	199	Add and subtract fractions and mixed numbers with like denominators (without regrouping) in number and word problems.
	202	Identify and draw angles (acute, right, obtuse, and straight).
	217	Draw and measure angles using a protractor. Understand that a circle measures 360 degrees.
	221	Find multiples, common multiples, and the least common multiple of numbers; explain.
	222	Find factors, common factors, and the greatest common factor of numbers; explain.
	223	Identify prime and composite numbers less than 100.
	266	Use remainders in problem-solving situations and interpret the remainder with respect to the original problem.
	540	Identify combinations of fractions that make one whole.
	607	Write a multiplication or a division sentence that represents a number or word problem; solve.
	660	Round whole numbers to a given place value.
	668	Write equivalent fractions with smaller or larger denominators.
	1012	Organize, display, and interpret information in line plots with a horizontal scale in fractional units.
	1013	Solve multi-step number and word problems using the four operations.
	1015	Add multi-digit numbers with regrouping in number and word problems.
	1016	Subtract multi-digit numbers with regrouping in number and word problems.
	1017	Use models to represent a fraction as a product of a whole number and a unit fraction in number and word problems.
	1023	Add and subtract fractions and mixed numbers using models and pictures to explain the process and record the results in number and word problems.
MOY	147	Determine the value of sets of coins and bills using cent sign and dollar sign appropriately. Create equivalent amounts with different coins and bills.
	154	Identify the place value of each digit in a multi-digit number to the thousandths place.
	164	Round decimals to a given place value; round fractions and mixed numbers to a whole number or a given fractional place value.
	170	Estimate and compute products of whole numbers with multi-digit factors.

Form	QSC	Description
	171	Estimate and solve division problems with multi-digit divisors; explain solution.
	173	Identify, draw, and name: points, rays, line segments, lines, and planes.
	176	Identify and draw intersecting, parallel, skew, and perpendicular lines and line segments. Identify midpoints of line segments.
	201	Estimate and compute sums and differences with decimals.
	202	Identify and draw angles (acute, right, obtuse, and straight).
	217	Draw and measure angles using a protractor. Understand that a circle measures 360 degrees.
	231	Add and subtract fractions and mixed numbers with unlike denominators in number and word problems.
	258	Convert measures of length, area, capacity, weight, and time expressed in a given unit to other units in the same measurement system in number and word problems.
	608	Multiply or divide two decimals or a decimal and a whole number in number and word problems.
	609	Multiply or divide with mixed numbers in number and word problems.
	675	Estimate sums and differences with fractions and mixed numbers.
	1012	Organize, display, and interpret information in line plots with a horizontal scale in fractional units.
	1014	Indicate and compare the place value of each digit in a multi-digit whole number or decimal.
	1018	Use models to develop the relationship between the total distance around a figure and the formula for perimeter; find perimeter using the formula in number and word problems.
	1019	Use addition and subtraction to find unknown measures of non-overlapping angles.
	1020	Translate between models or verbal phrases and numerical expressions.
	1021	Describe or compare the relationship between corresponding terms in two or more numerical patterns or tables of ratios.
	1024	Represent division of whole numbers as a fraction in number and word problems.
	1026	Represent division of a unit fraction by a whole number or a whole number by a unit fraction using models to explain the process in number and word problems.
EOY	160	Find the fractional part of a whole number or fraction with and without models and pictures.
	164	Round decimals to a given place value; round fractions and mixed numbers to a whole number or a given fractional place value.
	168	Describe the effect of operations on size and order of numbers.
	195	Read, write, and compare numbers with decimal place values to the thousandths place or numbers greater than one million.
	224	Multiply two fractions or a fraction and a whole number in number and word problems.
	230	Divide two fractions or a fraction and a whole number in number or word problems.

Form	QSC	Description
	231	Add and subtract fractions and mixed numbers with unlike denominators in number and word problems.
	258	Convert measures of length, area, capacity, weight, and time expressed in a given unit to other units in the same measurement system in number and word problems.
	547	Use a coordinate grid to solve number and word problems. Describe the path between given points on the plane.
	609	Multiply or divide with mixed numbers in number and word problems.
	620	Name polygons by the number of sides. Distinguish quadrilaterals based on properties of their sides or angles; relate quadrilaterals based upon their hierarchical attributes.
	630	Model the concept of the volume of a solid figure using cubic units.
	633	Recognize and use patterns in powers of ten (with or without exponents) to multiply and divide whole numbers and decimals.
	675	Estimate sums and differences with fractions and mixed numbers.
	1012	Organize, display, and interpret information in line plots with a horizontal scale in fractional units.
	1020	Translate between models or verbal phrases and numerical expressions.
	1021	Describe or compare the relationship between corresponding terms in two or more numerical patterns or tables of ratios.
	1023	Add and subtract fractions and mixed numbers using models and pictures to explain the process and record the results in number and word problems.
	1026	Represent division of a unit fraction by a whole number or a whole number by a unit fraction using models to explain the process in number and word problems.
	1027	Determine the volume of composite figures in number and word problems.

Grade 6

Form	QSC	Description
BOY	160	Find the fractional part of a whole number or fraction with and without models and pictures.
	164	Round decimals to a given place value; round fractions and mixed numbers to a whole number or a given fractional place value.
	168	Describe the effect of operations on size and order of numbers.
	195	Read, write, and compare numbers with decimal place values to the thousandths place or numbers greater than one million.
	231	Add and subtract fractions and mixed numbers with unlike denominators in number and word problems.
	243	Generate a set of ordered pairs using a rule which is stated in verbal, algebraic, or table form; generate a sequence given a rule in verbal or algebraic form.
	258	Convert measures of length, area, capacity, weight, and time expressed in a given unit to other units in the same measurement system in number and word problems.
	289	Use models to find volume for prisms and cylinders as the product of the area of the base (B) and the height. Calculate the volume of prisms in number and word problems.
	608	Multiply or divide two decimals or a decimal and a whole number in number and word problems.
	609	Multiply or divide with mixed numbers in number and word problems.
	620	Name polygons by the number of sides. Distinguish quadrilaterals based on properties of their sides or angles; relate quadrilaterals based upon their hierarchical attributes.
	630	Model the concept of the volume of a solid figure using cubic units.
	648	Read and write word names for rational numbers in decimal form to the hundredths place or the thousandths place.
	675	Estimate sums and differences with fractions and mixed numbers.
	1012	Organize, display, and interpret information in line plots with a horizontal scale in fractional units.
	1014	Indicate and compare the place value of each digit in a multi-digit whole number or decimal.
	1020	Translate between models or verbal phrases and numerical expressions.
	1022	Multiply and divide decimals using models and pictures to explain the process and record the results.
	1023	Add and subtract fractions and mixed numbers using models and pictures to explain the process and record the results in number and word problems.
	1024	Represent division of whole numbers as a fraction in number and word problems.
	1025	Represent multiplication of mixed numbers with and without models and pictures.
	1026	Represent division of a unit fraction by a whole number or a whole number by a unit fraction using models to explain the process in number and word problems.

Form	QSC	Description
	1027	Determine the volume of composite figures in number and word problems.
MOY	208	Solve one-step linear equations and inequalities and graph solutions of the inequalities on a number line in number and word problems.
	210	Write an equation to describe the algebraic relationship between two defined variables in number and word problems, including recognizing which variable is dependent.
	218	Translate between models or verbal phrases and algebraic expressions.
	222	Find factors, common factors, and the greatest common factor of numbers; explain.
	227	Relate a percent to its equivalent fraction or decimal.
	233	Calculate unit rates in number and word problems, including comparison of unit rates.
	235	Compare and order integers with and without models.
	236	Simplify numerical expressions that may contain exponents.
	244	Given a list of ordered pairs in a table or graph, identify either verbally or algebraically the rule used to generate and record the results.
	258	Convert measures of length, area, capacity, weight, and time expressed in a given unit to other units in the same measurement system in number and word problems.
	264	Calculate or estimate the percent of a number including discounts, taxes, commissions, and simple interest.
	274	Evaluate algebraic expressions in number and word problems.
	276	Write a linear equation or inequality to represent a given number or word problem; solve.
	289	Use models to find volume for prisms and cylinders as the product of the area of the base (B) and the height. Calculate the volume of prisms in number and word problems.
	300	Rewrite or simplify algebraic expressions including the use of the commutative, associative, and distributive properties, and inverses and identities in number and word problems.
	604	Graph or identify simple inequalities using symbol notation >, <, ≤, ≥, and ≠ in number and word problems.
	608	Multiply or divide two decimals or a decimal and a whole number in number and word problems.
	620	Name polygons by the number of sides. Distinguish quadrilaterals based on properties of their sides or angles; relate quadrilaterals based upon their hierarchical attributes.
	622	Solve number and word problems using percent proportion, percent equation, or ratios.
	624	Identify and classify triangles according to the measures of the interior angles and the lengths of the sides; relate triangles based upon their hierarchical attributes.
	636	Determine the absolute value of a number with and without models in number and word problems.
	1012	Organize, display, and interpret information in line plots with a horizontal scale in fractional units.

Form	QSC	Description
	1030	Contrast statements about absolute values of integers with statements about integer order.
	1032	Identify from a set of numbers which values satisfy a given equation or inequality.
EOY	171	Estimate and solve division problems with multi-digit divisors; explain solution.
	201	Estimate and compute sums and differences with decimals.
	208	Solve one-step linear equations and inequalities and graph solutions of the inequalities on a number line in number and word problems.
	210	Write an equation to describe the algebraic relationship between two defined variables in number and word problems, including recognizing which variable is dependent.
	214	Describe data using the mean.
	218	Translate between models or verbal phrases and algebraic expressions.
	221	Find multiples, common multiples, and the least common multiple of numbers; explain.
	222	Find factors, common factors, and the greatest common factor of numbers; explain.
	233	Calculate unit rates in number and word problems, including comparison of unit rates.
	235	Compare and order integers with and without models.
	236	Simplify numerical expressions that may contain exponents.
	244	Given a list of ordered pairs in a table or graph, identify either verbally or algebraically the rule used to generate and record the results.
	247	Locate points in all quadrants of the coordinate plane using ordered pairs in number and word problems.
	257	Calculate the areas of triangles, parallelograms, trapezoids, circles, and composite figures in number and word problems.
	260	Compare and order rational numbers with and without models.
	264	Calculate or estimate the percent of a number including discounts, taxes, commissions, and simple interest.
	276	Write a linear equation or inequality to represent a given number or word problem; solve.
	310	Organize, display, and interpret information in box-and-whisker plots.
	318	Use nets or formulas to find the surface area of prisms, pyramids, and cylinders in number and word problems.
	547	Use a coordinate grid to solve number and word problems. Describe the path between given points on the plane.
	609	Multiply or divide with mixed numbers in number and word problems.
	622	Solve number and word problems using percent proportion, percent equation, or ratios.
	636	Determine the absolute value of a number with and without models in number and word problems.
	1030	Contrast statements about absolute values of integers with statements about integer order.

Form	QSC	Description
	1032	Identify from a set of numbers which values satisfy a given equation or inequality.
	1033	Recognize that a statistical question is one that will require gathering data that has variability.

Grade 7

Form	QSC	Description
BOY	169	Use concepts of positive numbers, negative numbers, and zero (e.g., on a number line, in counting, in temperature, in "owing") to describe quantities in number and word problems.
	208	Solve one-step linear equations and inequalities and graph solutions of the inequalities on a number line in number and word problems.
	210	Write an equation to describe the algebraic relationship between two defined variables in number and word problems, including recognizing which variable is dependent.
	218	Translate between models or verbal phrases and algebraic expressions.
	219	Estimate the measure of an object in one system given the measure of that object in another system.
	220	Use exponential notation and repeated multiplication to describe and simplify exponential expressions.
	221	Find multiples, common multiples, and the least common multiple of numbers; explain.
	222	Find factors, common factors, and the greatest common factor of numbers; explain.
	230	Divide two fractions or a fraction and a whole number in number or word problems.
	233	Calculate unit rates in number and word problems, including comparison of unit rates.
	235	Compare and order integers with and without models.
	244	Given a list of ordered pairs in a table or graph, identify either verbally or algebraically the rule used to generate and record the results.
	247	Locate points in all quadrants of the coordinate plane using ordered pairs in number and word problems.
	257	Calculate the areas of triangles, parallelograms, trapezoids, circles, and composite figures in number and word problems.
	258	Convert measures of length, area, capacity, weight, and time expressed in a given unit to other units in the same measurement system in number and word problems.
	264	Calculate or estimate the percent of a number including discounts, taxes, commissions, and simple interest.
	276	Write a linear equation or inequality to represent a given number or word problem; solve.
	289	Use models to find volume for prisms and cylinders as the product of the area of the base (B) and the height. Calculate the volume of prisms in number and word problems.
	300	Rewrite or simplify algebraic expressions including the use of the commutative, associative, and distributive properties, and inverses and identities in number and word problems.
	310	Organize, display, and interpret information in box-and-whisker plots.
	559	Determine the quartiles or interquartile range for a set of data.
	609	Multiply or divide with mixed numbers in number and word problems.

Form	QSC	Description
	622	Solve number and word problems using percent proportion, percent equation, or ratios.
	1021	Describe or compare the relationship between corresponding terms in two or more numerical patterns or tables of ratios.
	1032	Identify from a set of numbers which values satisfy a given equation or inequality.
	1036	Select the appropriate measure of variability; choose a measure of variability based on the presence of outliers, clusters, and the shape of the data distribution.
MOY	196	Convert fractions and terminating decimals to the thousandths place to equivalent forms without models; explain the equivalence.
	247	Locate points in all quadrants of the coordinate plane using ordered pairs in number and word problems.
	256	Use models to develop formulas for finding areas of triangles, parallelograms, trapezoids, and circles in number and word problems.
	261	Model or compute with integers using addition or subtraction in number and word problems.
	262	Model or compute with integers using multiplication or division in number and word problems.
	264	Calculate or estimate the percent of a number including discounts, taxes, commissions, and simple interest.
	275	Solve two-step linear equations and inequalities and graph solutions of the inequalities on a number line.
	276	Write a linear equation or inequality to represent a given number or word problem; solve.
	278	Organize, display, and interpret information in histograms.
	289	Use models to find volume for prisms and cylinders as the product of the area of the base (B) and the height. Calculate the volume of prisms in number and word problems.
	295	Solve number and word problems involving percent increase and percent decrease.
	300	Rewrite or simplify algebraic expressions including the use of the commutative, associative, and distributive properties, and inverses and identities in number and word problems.
	310	Organize, display, and interpret information in box-and-whisker plots.
	318	Use nets or formulas to find the surface area of prisms, pyramids, and cylinders in number and word problems.
	332	Solve linear equations using the associative, commutative, distributive, and equality properties and justify the steps used.
	362	Write equations to represent direct variation and use direct variation to solve number and word problems.
	642	Perform multi-step operations with rational numbers (positive and negative) in number and word problems.
	644	Model and solve linear inequalities using the properties of inequality in number and word problems.
	669	Estimate products and quotients of decimals or of mixed numbers.
	1033	Recognize that a statistical question is one that will require gathering data that has variability.

Form	QSC	Description
	1034	Use frequency tables, dot plots, and other graphs to determine the shape, center, and spread of a data distribution.
	1037	Calculate unit rates of ratios that include fractions to make comparisons in number and word problems.
	1038	Given a proportional relationship represented by tables, graphs, models, or algebraic or verbal descriptions, identify the unit rate (constant of proportionality).
	1039	Use the commutative, associative, and distributive properties, and inverses and identities to solve number and word problems with rational numbers.
	1040	Use the definition of rational numbers to convert decimals and fractions to equivalent forms.
EOY	196	Convert fractions and terminating decimals to the thousandths place to equivalent forms without models; explain the equivalence.
	239	Define and identify complementary and supplementary angles.
	240	Define and identify alternate interior, alternate exterior, corresponding, adjacent and vertical angles.
	251	Determine the sample space for an event using counting strategies (include tree diagrams, permutations, combinations, and the Fundamental Counting Principle).
	254	Investigate and determine the relationship between the diameter and the circumference of a circle and the value of pi; calculate the circumference of a circle.
	257	Calculate the areas of triangles, parallelograms, trapezoids, circles, and composite figures in number and word problems.
	261	Model or compute with integers using addition or subtraction in number and word problems.
	262	Model or compute with integers using multiplication or division in number and word problems.
	275	Solve two-step linear equations and inequalities and graph solutions of the inequalities on a number line.
	276	Write a linear equation or inequality to represent a given number or word problem; solve.
	285	Determine the probability of compound events (with and without replacement).
	295	Solve number and word problems involving percent increase and percent decrease.
	300	Rewrite or simplify algebraic expressions including the use of the commutative, associative, and distributive properties, and inverses and identities in number and word problems.
	314	Distinguish between a population and a sample and draw conclusions about the sample (random or biased).
	317	Calculate distances from scale drawings and maps.
	556	Describe cross-sectional views of three-dimensional figures.
	562	Use ordered pairs derived from tables, algebraic rules, or verbal descriptions to graph linear functions.
	567	Identify relations as directly proportional, linear, or nonlinear using rules, tables, and graphs.
	642	Perform multi-step operations with rational numbers (positive and negative) in number and word problems.

Form	**QSC**	**Description**
	644	Model and solve linear inequalities using the properties of inequality in number and word problems.
	1037	Calculate unit rates of ratios that include fractions to make comparisons in number and word problems.
	1038	Given a proportional relationship represented by tables, graphs, models, or algebraic or verbal descriptions, identify the unit rate (constant of proportionality).
	1039	Use the commutative, associative, and distributive properties, and inverses and identities to solve number and word problems with rational numbers.
	1040	Use the definition of rational numbers to convert decimals and fractions to equivalent forms.
	1043	Make predictions based on results from surveys and samples.
	1044	Compare data and distributions of data, numerical and contextual, to draw conclusions, considering the measures of center and measures of variability.

Grade 8

Form	QSC	Description
BOY	196	Convert fractions and terminating decimals to the thousandths place to equivalent forms without models; explain the equivalence.
	239	Define and identify complementary and supplementary angles.
	251	Determine the sample space for an event using counting strategies (include tree diagrams, permutations, combinations, and the Fundamental Counting Principle).
	256	Use models to develop formulas for finding areas of triangles, parallelograms, trapezoids, and circles in number and word problems.
	257	Calculate the areas of triangles, parallelograms, trapezoids, circles, and composite figures in number and word problems.
	261	Model or compute with integers using addition or subtraction in number and word problems.
	262	Model or compute with integers using multiplication or division in number and word problems.
	263	Write a proportion to model a word problem; solve proportions.
	264	Calculate or estimate the percent of a number including discounts, taxes, commissions, and simple interest.
	275	Solve two-step linear equations and inequalities and graph solutions of the inequalities on a number line.
	276	Write a linear equation or inequality to represent a given number or word problem; solve.
	285	Determine the probability of compound events (with and without replacement).
	295	Solve number and word problems involving percent increase and percent decrease.
	323	Evaluate absolute value expressions to solve number and word problems, including finding horizontal and vertical distances between points and lines.
	556	Describe cross-sectional views of three-dimensional figures.
	567	Identify relations as directly proportional, linear, or nonlinear using rules, tables, and graphs.
	642	Perform multi-step operations with rational numbers (positive and negative) in number and word problems.
	1037	Calculate unit rates of ratios that include fractions to make comparisons in number and word problems.
	1038	Given a proportional relationship represented by tables, graphs, models, or algebraic or verbal descriptions, identify the unit rate (constant of proportionality).
	1040	Use the definition of rational numbers to convert decimals and fractions to equivalent forms.
	1041	Recognize conditions of side lengths that determine a unique triangle, more than one triangle, or no triangle.
	1043	Make predictions based on results from surveys and samples.
	1044	Compare data and distributions of data, numerical and contextual, to draw conclusions, considering the measures of center and measures of variability.

Form	QSC	Description
MOY	209	Analyze graphs, identify situations, or solve problems with varying rates of change.
	239	Define and identify complementary and supplementary angles.
	249	Determine the probability from experimental results or compare theoretical probabilities and experimental results.
	251	Determine the sample space for an event using counting strategies (include tree diagrams, permutations, combinations, and the Fundamental Counting Principle).
	254	Investigate and determine the relationship between the diameter and the circumference of a circle and the value of pi; calculate the circumference of a circle.
	259	Write whole numbers in scientific notation; convert scientific notation to standard form; investigate the uses of scientific notation.
	289	Use models to find volume for prisms and cylinders as the product of the area of the base (B) and the height. Calculate the volume of prisms in number and word problems.
	296	Use rules of exponents to simplify numeric and algebraic expressions.
	298	Estimate and calculate using numbers expressed in scientific notation.
	307	Identify and interpret the intercepts of a linear relation in number and word problems.
	309	Graphically solve systems of linear equations.
	314	Distinguish between a population and a sample and draw conclusions about the sample (random or biased).
	318	Use nets or formulas to find the surface area of prisms, pyramids, and cylinders in number and word problems.
	330	Define and distinguish between relations and functions, dependent and independent variables, and domain and range; identify whether relations are functions numerically and graphically.
	332	Solve linear equations using the associative, commutative, distributive, and equality properties and justify the steps used.
	333	Write and solve systems of linear equations in two or more variables algebraically in number and word problems.
	344	Describe the slope of a line given in the context of a problem situation; compare rates of change in linear relationships represented in different ways.
	345	Write the equation of and graph linear relationships given the slope and y-intercept.
	556	Describe cross-sectional views of three-dimensional figures.
	562	Use ordered pairs derived from tables, algebraic rules, or verbal descriptions to graph linear functions.
	564	Describe, use, and compare real numbers. Use the definition of rational numbers to derive and distinguish irrational numbers.
	1040	Use the definition of rational numbers to convert decimals and fractions to equivalent forms.
	1043	Make predictions based on results from surveys and samples.
	1046	Interpret probability models for data from simulations or for experimental data presented in tables and graphs (frequency tables, line plots, bar graphs).

Form	QSC	Description
	1049	Determine whether a linear equation has one solution, infinitely many solutions, or no solution.
EOY	209	Analyze graphs, identify situations, or solve problems with varying rates of change.
	259	Write whole numbers in scientific notation; convert scientific notation to standard form; investigate the uses of scientific notation.
	270	Locate, given the coordinates of, and graph plane figures which are the results of translations or reflections in all quadrants of the coordinate plane.
	287	Determine and use scale factors to reduce and enlarge drawings on grids to produce dilations.
	292	Use proportions to express relationships between corresponding parts of similar figures.
	296	Use rules of exponents to simplify numeric and algebraic expressions.
	298	Estimate and calculate using numbers expressed in scientific notation.
	302	Use the Pythagorean Theorem and its converse to solve number and word problems, including finding the distance between two points.
	303	Locate, given the coordinates of, and graph plane figures which are the results of rotations (multiples of 90 degrees) with respect to a given point.
	307	Identify and interpret the intercepts of a linear relation in number and word problems.
	309	Graphically solve systems of linear equations.
	320	Calculate the volume of cylinders, pyramids, and cones in number and word problems.
	330	Define and distinguish between relations and functions, dependent and independent variables, and domain and range; identify whether relations are functions numerically and graphically.
	332	Solve linear equations using the associative, commutative, distributive, and equality properties and justify the steps used.
	333	Write and solve systems of linear equations in two or more variables algebraically in number and word problems.
	492	Use definitions and theorems of angles formed when a transversal intersects parallel lines.
	562	Use ordered pairs derived from tables, algebraic rules, or verbal descriptions to graph linear functions.
	564	Describe, use, and compare real numbers. Use the definition of rational numbers to derive and distinguish irrational numbers.
	565	Approximate a linear model that best fits a set of data; use the linear model to make predictions.
	567	Identify relations as directly proportional, linear, or nonlinear using rules, tables, and graphs.
	568	Interpret and compare properties of linear functions, graphs, and equations.
	572	Identify linear and nonlinear relationships in data sets.
	1040	Use the definition of rational numbers to convert decimals and fractions to equivalent forms.

Form	QSC	Description
	1049	Determine whether a linear equation has one solution, infinitely many solutions, or no solution.
	1053	Identify outliers and clusters in bivariate data in tables and scatter plots.

Grade 9

Form	QSC	Description
BOY	209	Analyze graphs, identify situations, or solve problems with varying rates of change.
	259	Write whole numbers in scientific notation; convert scientific notation to standard form; investigate the uses of scientific notation.
	270	Locate, given the coordinates of, and graph plane figures which are the results of translations or reflections in all quadrants of the coordinate plane.
	287	Determine and use scale factors to reduce and enlarge drawings on grids to produce dilations.
	296	Use rules of exponents to simplify numeric and algebraic expressions.
	298	Estimate and calculate using numbers expressed in scientific notation.
	302	Use the Pythagorean Theorem and its converse to solve number and word problems, including finding the distance between two points.
	309	Graphically solve systems of linear equations.
	311	Organize, display, and interpret information in scatter plots. Approximate a trend line and identify the relationship as positive, negative, or no correlation.
	320	Calculate the volume of cylinders, pyramids, and cones in number and word problems.
	330	Define and distinguish between relations and functions, dependent and independent variables, and domain and range; identify whether relations are functions numerically and graphically.
	332	Solve linear equations using the associative, commutative, distributive, and equality properties and justify the steps used.
	333	Write and solve systems of linear equations in two or more variables algebraically in number and word problems.
	344	Describe the slope of a line given in the context of a problem situation; compare rates of change in linear relationships represented in different ways.
	347	Write the equation of and graph linear relationships given two points on the line.
	492	Use definitions and theorems of angles formed when a transversal intersects parallel lines.
	562	Use ordered pairs derived from tables, algebraic rules, or verbal descriptions to graph linear functions.
	564	Describe, use, and compare real numbers. Use the definition of rational numbers to derive and distinguish irrational numbers.
	565	Approximate a linear model that best fits a set of data; use the linear model to make predictions.
	568	Interpret and compare properties of linear functions, graphs, and equations.
	1040	Use the definition of rational numbers to convert decimals and fractions to equivalent forms.
	1049	Determine whether a linear equation has one solution, infinitely many solutions, or no solution.

Form	QSC	Description
	1050	Verify how properties and relationships of geometric figures are maintained or how they change through transformations.
	1052	Calculate the surface area and volume of a sphere in number and word problems.
	1054	Construct and interpret a two-way table to display two categories of data from the same source.
MOY	210	Write an equation to describe the algebraic relationship between two defined variables in number and word problems, including recognizing which variable is dependent.
	259	Write whole numbers in scientific notation; convert scientific notation to standard form; investigate the uses of scientific notation.
	270	Locate, given the coordinates of, and graph plane figures which are the results of translations or reflections in all quadrants of the coordinate plane.
	276	Write a linear equation or inequality to represent a given number or word problem; solve.
	287	Determine and use scale factors to reduce and enlarge drawings on grids to produce dilations.
	302	Use the Pythagorean Theorem and its converse to solve number and word problems, including finding the distance between two points.
	303	Locate, given the coordinates of, and graph plane figures which are the results of rotations (multiples of 90 degrees) with respect to a given point.
	306	Determine algebraically or graphically the solutions of a linear inequality in two variables.
	307	Identify and interpret the intercepts of a linear relation in number and word problems.
	311	Organize, display, and interpret information in scatter plots. Approximate a trend line and identify the relationship as positive, negative, or no correlation.
	320	Calculate the volume of cylinders, pyramids, and cones in number and word problems.
	332	Solve linear equations using the associative, commutative, distributive, and equality properties and justify the steps used.
	333	Write and solve systems of linear equations in two or more variables algebraically in number and word problems.
	346	Write the equation of and graph linear relationships given the slope and one point on the line.
	347	Write the equation of and graph linear relationships given two points on the line.
	350	Determine the effects of changes in slope and/or intercepts on graphs and equations of lines.
	365	Describe graphically, algebraically and verbally real-world phenomena as functions; identify the independent and dependent variables and any constraints of the domain or range.
	366	Convert between different representations of relations and functions using tables, the coordinate plane, and algebraic or verbal statements.
	400	Write the equation of and graph exponential equations or functions, including f(x) = ab^x and f(x) = a(1+r)^x, in number and word problems; identify and interpret critical values.

Form	QSC	Description
	562	Use ordered pairs derived from tables, algebraic rules, or verbal descriptions to graph linear functions.
	564	Describe, use, and compare real numbers. Use the definition of rational numbers to derive and distinguish irrational numbers.
	631	Use rules of exponents to rewrite or simplify expressions with rational exponents or radicals and interpret their meaning.
	656	Recognize and extend arithmetic sequences and geometric sequences. Identify the common difference or common ratio.
	659	Solve a literal equation for an indicated variable.
	1051	Describe, or graph plane figures which are the results of a sequence of transformations.
	1052	Calculate the surface area and volume of a sphere in number and word problems.
	1053	Identify outliers and clusters in bivariate data in tables and scatter plots.
	1054	Construct and interpret a two-way table to display two categories of data from the same source.
	1058	Determine whether a system of equations has one solution, multiple solutions, infinitely many solutions, or no solution using graphs, tables, and algebraic methods; compare solutions of systems of equations.
	1061	Classify functions as linear, quadratic, rational, etc. based on their tabular, graphical, verbal, or algebraic description; compare properties of two or more function represented in different ways.
EOY	270	Locate, given the coordinates of, and graph plane figures which are the results of translations or reflections in all quadrants of the coordinate plane.
	296	Use rules of exponents to simplify numeric and algebraic expressions.
	310	Organize, display, and interpret information in box-and-whisker plots.
	322	Determine precision unit, accuracy, and greatest possible error of a measuring tool. Apply significant digits in meaningful contexts.
	332	Solve linear equations using the associative, commutative, distributive, and equality properties and justify the steps used.
	335	Graph quadratic functions. Identify and interpret the intercepts, maximum, minimum, and the axis of symmetry.
	342	Derive a linear equation that models a set of data (line of best fit) using calculators. Use the model to make predictions.
	346	Write the equation of and graph linear relationships given the slope and one point on the line.
	347	Write the equation of and graph linear relationships given two points on the line.
	350	Determine the effects of changes in slope and/or intercepts on graphs and equations of lines.
	365	Describe graphically, algebraically and verbally real-world phenomena as functions; identify the independent and dependent variables and any constraints of the domain or range.
	477	Determine measures of spread (standard deviation).

Form	QSC	Description
	564	Describe, use, and compare real numbers. Use the definition of rational numbers to derive and distinguish irrational numbers.
	574	Recognize and apply algebra techniques to solve rate problems including distance, work, density, and mixture problems.
	631	Use rules of exponents to rewrite or simplify expressions with rational exponents or radicals and interpret their meaning.
	644	Model and solve linear inequalities using the properties of inequality in number and word problems.
	656	Recognize and extend arithmetic sequences and geometric sequences. Identify the common difference or common ratio.
	659	Solve a literal equation for an indicated variable.
	671	Use dimensional analysis to rename quantities or rates.
	674	Solve systems of linear inequalities.
	1044	Compare data and distributions of data, numerical and contextual, to draw conclusions, considering the measures of center and measures of variability.
	1051	Describe, or graph plane figures which are the results of a sequence of transformations.
	1055	Determine and interpret the components of algebraic expressions including terms, factors, variables, coefficients, constants, and parts of powers in number and word problems.
	1056	Use appropriate units to model, solve, and estimate multi-step word problems.
	1058	Determine whether a system of equations has one solution, multiple solutions, infinitely many solutions, or no solution using graphs, tables, and algebraic methods; compare solutions of systems of equations.
	1061	Classify functions as linear, quadratic, rational, etc. based on their tabular, graphical, verbal, or algebraic description; compare properties of two or more function represented in different ways.
	1069	Express data in a two-way table. Calculate marginal distribution, marginal and conditional probabilities, or basic probabilities.
	1070	Determine or calculate residuals of a distribution.

Grade 10

Form	QSC	Description
BOY	277	Write a problem given a simple linear equation or inequality.
	296	Use rules of exponents to simplify numeric and algebraic expressions.
	306	Determine algebraically or graphically the solutions of a linear inequality in two variables.
	307	Identify and interpret the intercepts of a linear relation in number and word problems.
	310	Organize, display, and interpret information in box-and-whisker plots.
	333	Write and solve systems of linear equations in two or more variables algebraically in number and word problems.
	346	Write the equation of and graph linear relationships given the slope and one point on the line.
	347	Write the equation of and graph linear relationships given two points on the line.
	350	Determine the effects of changes in slope and/or intercepts on graphs and equations of lines.
	365	Describe graphically, algebraically and verbally real-world phenomena as functions; identify the independent and dependent variables and any constraints of the domain or range.
	366	Convert between different representations of relations and functions using tables, the coordinate plane, and algebraic or verbal statements.
	477	Determine measures of spread (standard deviation).
	490	Use properties, definitions, and theorems of angles and lines to solve problems related to the segment addition postulate and the angle addition postulate.
	564	Describe, use, and compare real numbers. Use the definition of rational numbers to derive and distinguish irrational numbers.
	565	Approximate a linear model that best fits a set of data; use the linear model to make predictions.
	631	Use rules of exponents to rewrite or simplify expressions with rational exponents or radicals and interpret their meaning.
	644	Model and solve linear inequalities using the properties of inequality in number and word problems.
	656	Recognize and extend arithmetic sequences and geometric sequences. Identify the common difference or common ratio.
	659	Solve a literal equation for an indicated variable.
	671	Use dimensional analysis to rename quantities or rates.
	1044	Compare data and distributions of data, numerical and contextual, to draw conclusions, considering the measures of center and measures of variability.
	1051	Describe, or graph plane figures which are the results of a sequence of transformations.
	1055	Determine and interpret the components of algebraic expressions including terms, factors, variables, coefficients, constants, and parts of powers in number and word problems.

Form	QSC	Description
	1056	Use appropriate units to model, solve, and estimate multi-step word problems.
	1058	Determine whether a system of equations has one solution, multiple solutions, infinitely many solutions, or no solution using graphs, tables, and algebraic methods; compare solutions of systems of equations.
	1061	Classify functions as linear, quadratic, rational, etc. based on their tabular, graphical, verbal, or algebraic description; compare properties of two or more function represented in different ways.
	1069	Express data in a two-way table. Calculate marginal distribution, marginal and conditional probabilities, or basic probabilities.
MOY	270	Locate, given the coordinates of, and graph plane figures which are the results of translations or reflections in all quadrants of the coordinate plane.
	303	Locate, given the coordinates of, and graph plane figures which are the results of rotations (multiples of 90 degrees) with respect to a given point.
	310	Organize, display, and interpret information in box-and-whisker plots.
	355	Perform basic operations with complex numbers and graph complex numbers.
	477	Determine measures of spread (standard deviation).
	483	Use the distance formula to find the distance between two points. Use the midpoint formula to find the coordinates of the midpoint of a segment.
	492	Use definitions and theorems of angles formed when a transversal intersects parallel lines.
	497	Use properties, definitions, and theorems to determine the congruency or similarity of polygons in order to solve problems.
	499	Use coordinate geometry to confirm properties of plane figures.
	504	Use properties of triangles to solve problems related to congruent triangles and their corresponding parts.
	518	Use properties of circles to solve problems related to the equation of a circle, its center, and radius length.
	519	Use pictorial representations and appropriate vocabulary to identify relationships with circles (e.g. tangent, secant, concentric circles, inscribe, circumscribe, semicircles, and minor and major arcs) in number and word problems.
	532	Use slopes to determine if two lines are parallel or perpendicular.
	534	Transform (translate, reflect, rotate, dilate) polygons in the coordinate plane; describe the transformation in simple algebraic terms.
	565	Approximate a linear model that best fits a set of data; use the linear model to make predictions.
	595	Describe and simplify imaginary and complex numbers.
	631	Use rules of exponents to rewrite or simplify expressions with rational exponents or radicals and interpret their meaning.
	658	Complete the square to identify characteristics of relations or functions and verify graphically.
	671	Use dimensional analysis to rename quantities or rates.

Form	QSC	Description
	673	Use the definition of a parabola to identify characteristics, write an equation, and graph the relation.
	1044	Compare data and distributions of data, numerical and contextual, to draw conclusions, considering the measures of center and measures of variability.
	1050	Verify how properties and relationships of geometric figures are maintained or how they change through transformations.
	1051	Describe, or graph plane figures which are the results of a sequence of transformations.
	1054	Construct and interpret a two-way table to display two categories of data from the same source.
	1056	Use appropriate units to model, solve, and estimate multi-step word problems.
	1070	Determine or calculate residuals of a distribution.
	1072	Use set notation to describe domains, ranges, intersection, and union of sets. Identify cardinality of sets, equivalent sets, disjoint sets, complement, or subsets.
EOY	319	Use models to investigate the relationship of the volume of a cone to a cylinder and a pyramid to a prism with the same base and height.
	321	Determine the effect on the volume of solid figures when one or more dimension is changed.
	355	Perform basic operations with complex numbers and graph complex numbers.
	424	Determine the measure of an angle in degree mode or in radian mode.
	489	Use properties, definitions, and theorems of angles and lines to solve problems related to adjacent, vertical, complementary, supplementary, and linear pairs of angles.
	491	Use properties, definitions, and theorems of angles and lines to solve problems related to angle bisectors, segment bisectors, and perpendicular bisectors.
	497	Use properties, definitions, and theorems to determine the congruency or similarity of polygons in order to solve problems.
	499	Use coordinate geometry to confirm properties of plane figures.
	500	Use properties, definitions, and theorems of quadrilaterals (parallelograms, rectangles, rhombi, squares, trapezoids, kites) to solve problems.
	503	Use properties of triangles to solve problems related to similar triangles and the relationships of their corresponding parts.
	504	Use properties of triangles to solve problems related to congruent triangles and their corresponding parts.
	505	Use properties of triangles to solve problems related to isosceles and equilateral triangles.
	506	Use properties of triangles to solve problems related to altitudes, perpendicular bisectors, angle bisectors, and medians.
	514	Use properties of right triangles to solve problems using the relationships in special right triangles.
	515	Use trigonometric ratios to represent relationships in right triangles to solve number and word problems.

Form	QSC	Description
	518	Use properties of circles to solve problems related to the equation of a circle, its center, and radius length.
	519	Use pictorial representations and appropriate vocabulary to identify relationships with circles (e.g. tangent, secant, concentric circles, inscribe, circumscribe, semicircles, and minor and major arcs) in number and word problems.
	523	Use properties of circles to solve number and word problems involving arcs formed by central angles or inscribed angles.
	595	Describe and simplify imaginary and complex numbers.
	652	Use theorems about arc measures determined by rays of angles formed by two lines intersecting a circle when the vertex is inside the circle (two chords), on the circle (tangent and chord), or outside the circle (two secants).
	673	Use the definition of a parabola to identify characteristics, write an equation, and graph the relation.
	1069	Express data in a two-way table. Calculate marginal distribution, marginal and conditional probabilities, or basic probabilities.
	1071	Use various methods, including trigonometric relationships or Heron's Formula, to find the area of a triangle in number and word problems.
	1072	Use set notation to describe domains, ranges, intersection, and union of sets. Identify cardinality of sets, equivalent sets, disjoint sets, complement, or subsets.
	1073	Distinguish between types of events (conditional, mutually exclusive, independent, dependent, etc.). Use the appropriate formula to determine probabilities of random phenomena using addition rule, multiplication rule, or Venn diagrams.
	1074	Compare theoretical probabilities to results from any simulations or experiments (may also include Law of Large Numbers).

Grade 11/12

Form	QSC	Description
BOY	292	Use proportions to express relationships between corresponding parts of similar figures.
	321	Determine the effect on the volume of solid figures when one or more dimension is changed.
	424	Determine the measure of an angle in degree mode or in radian mode.
	489	Use properties, definitions, and theorems of angles and lines to solve problems related to adjacent, vertical, complementary, supplementary, and linear pairs of angles.
	491	Use properties, definitions, and theorems of angles and lines to solve problems related to angle bisectors, segment bisectors, and perpendicular bisectors.
	499	Use coordinate geometry to confirm properties of plane figures.
	500	Use properties, definitions, and theorems of quadrilaterals (parallelograms, rectangles, rhombi, squares, trapezoids, kites) to solve problems.
	503	Use properties of triangles to solve problems related to similar triangles and the relationships of their corresponding parts.
	506	Use properties of triangles to solve problems related to altitudes, perpendicular bisectors, angle bisectors, and medians.
	509	Use properties of triangles to solve problems related to the segments parallel to one side of a triangle, including segments joining the midpoints of two sides of a triangle (midsegments).
	512	Use properties of right triangles to solve problems using the geometric mean.
	515	Use trigonometric ratios to represent relationships in right triangles to solve number and word problems.
	518	Use properties of circles to solve problems related to the equation of a circle, its center, and radius length.
	519	Use pictorial representations and appropriate vocabulary to identify relationships with circles (e.g. tangent, secant, concentric circles, inscribe, circumscribe, semicircles, and minor and major arcs) in number and word problems.
	523	Use properties of circles to solve number and word problems involving arcs formed by central angles or inscribed angles.
	529	Use measures of arcs or central angles to find arc length or sector area of a circle.
	595	Describe and simplify imaginary and complex numbers.
	631	Use rules of exponents to rewrite or simplify expressions with rational exponents or radicals and interpret their meaning.
	652	Use theorems about arc measures determined by rays of angles formed by two lines intersecting a circle when the vertex is inside the circle (two chords), on the circle (tangent and chord), or outside the circle (two secants).
	673	Use the definition of a parabola to identify characteristics, write an equation, and graph the relation.
	1063	Use properties, definitions, and theorems to solve problems about rigid transformations and dilations of plane figures.

Form	QSC	Description
	1069	Express data in a two-way table. Calculate marginal distribution, marginal and conditional probabilities, or basic probabilities.
	1072	Use set notation to describe domains, ranges, intersection, and union of sets. Identify cardinality of sets, equivalent sets, disjoint sets, complement, or subsets.
	1073	Distinguish between types of events (conditional, mutually exclusive, independent, dependent, etc.). Use the appropriate formula to determine probabilities of random phenomena using addition rule, multiplication rule, or Venn diagrams.
	1074	Compare theoretical probabilities to results from any simulations or experiments (may also include Law of Large Numbers).
MOY	287	Determine and use scale factors to reduce and enlarge drawings on grids to produce dilations.
	321	Determine the effect on the volume of solid figures when one or more dimension is changed.
	325	Add, subtract, and multiply polynomials.
	326	Divide polynomials by monomial divisors.
	327	Factor quadratic polynomials, including special products.
	333	Write and solve systems of linear equations in two or more variables algebraically in number and word problems.
	335	Graph quadratic functions. Identify and interpret the intercepts, maximum, minimum, and the axis of symmetry.
	339	Evaluate expressions and use formulas to solve number and word problems involving exponential functions; classify exponential functions as exponential growth or decay.
	375	Find and interpret the maximum, the minimum, and the intercepts of a quadratic function.
	383	Identify asymptotes, intercepts, holes, domain, and range of a rational function and sketch the graph.
	398	Graph absolute value functions and their corresponding inequalities.
	479	Define and use the normal distribution curve to model a set of data; estimate the area under the curve.
	492	Use definitions and theorems of angles formed when a transversal intersects parallel lines.
	500	Use properties, definitions, and theorems of quadrilaterals (parallelograms, rectangles, rhombi, squares, trapezoids, kites) to solve problems.
	509	Use properties of triangles to solve problems related to the segments parallel to one side of a triangle, including segments joining the midpoints of two sides of a triangle (midsegments).
	529	Use measures of arcs or central angles to find arc length or sector area of a circle.
	595	Describe and simplify imaginary and complex numbers.
	631	Use rules of exponents to rewrite or simplify expressions with rational exponents or radicals and interpret their meaning.
	652	Use theorems about arc measures determined by rays of angles formed by two lines intersecting a circle when the vertex is inside the circle (two chords), on the circle (tangent and chord), or outside the circle (two secants).

Form	QSC	Description
	672	Determine the area and volume of figures using right triangle relationships, including trigonometric relationships in number and word problems.
	1060	Given a specific interval, find the average rate of change of a function using a table, graph, or an algebraic description.
	1063	Use properties, definitions, and theorems to solve problems about rigid transformations and dilations of plane figures.
	1066	Factor a polynomial using grouping techniques by recognizing quadratic form and forms of special products, including factors with complex numbers.
	1067	Find algebraically or approximate graphically or numerically solutions of equations of the form f(x) = g(x) where f(x) and g(x) are linear, polynomial, rational, radical, absolute value, exponential, logarithmic, or trigonometric functions.
	1069	Express data in a two-way table. Calculate marginal distribution, marginal and conditional probabilities, or basic probabilities.
	1072	Use set notation to describe domains, ranges, intersection, and union of sets. Identify cardinality of sets, equivalent sets, disjoint sets, complement, or subsets.
	1073	Distinguish between types of events (conditional, mutually exclusive, independent, dependent, etc.). Use the appropriate formula to determine probabilities of random phenomena using addition rule, multiplication rule, or Venn diagrams.
	1074	Compare theoretical probabilities to results from any simulations or experiments (may also include Law of Large Numbers).
EOY	325	Add, subtract, and multiply polynomials.
	326	Divide polynomials by monomial divisors.
	333	Write and solve systems of linear equations in two or more variables algebraically in number and word problems.
	335	Graph quadratic functions. Identify and interpret the intercepts, maximum, minimum, and the axis of symmetry.
	336	Identify and interpret zeros of a quadratic function using factoring in algebraic and word problems.
	339	Evaluate expressions and use formulas to solve number and word problems involving exponential functions; classify exponential functions as exponential growth or decay.
	350	Determine the effects of changes in slope and/or intercepts on graphs and equations of lines.
	355	Perform basic operations with complex numbers and graph complex numbers.
	358	Divide one polynomial by another of a lower degree using either synthetic division or the division algorithm.
	384	Write and solve rational equations; identify extraneous solutions, including checking the solution in the original equation.
	388	Graph a radical relation, function, or inequality. State the domain and range.
	389	Write and solve radical equations and inequalities; identify extraneous solutions, including checking the solution in the original equation.
	398	Graph absolute value functions and their corresponding inequalities.

Form	QSC	Description
	400	Write the equation of and graph exponential equations or functions, including f(x) = ab^x and f(x) = a(1+r)^x, in number and word problems; identify and interpret critical values.
	420	Use a unit circle to define trigonometric functions and evaluate trigonometric functions for a given angle.
	422	Use the unit circle to define and validate trigonometric identities.
	424	Determine the measure of an angle in degree mode or in radian mode.
	432	Describe and use the symmetry of a graph and determine whether a function is even, odd, or neither.
	595	Describe and simplify imaginary and complex numbers.
	631	Use rules of exponents to rewrite or simplify expressions with rational exponents or radicals and interpret their meaning.
	659	Solve a literal equation for an indicated variable.
	662	Write a quadratic equation or quadratic function given its zeros.
	1060	Given a specific interval, find the average rate of change of a function using a table, graph, or an algebraic description.
	1062	Recognize closure of number systems under a collection of operations and their properties with and without models; extend closure to analogous algebraic systems.
	1066	Factor a polynomial using grouping techniques by recognizing quadratic form and forms of special products, including factors with complex numbers.
	1074	Compare theoretical probabilities to results from any simulations or experiments (may also include Law of Large Numbers).
	1075	Examine, interpret, or apply probability or game theory strategies to determine the fairness of outcomes of various situations, including games, economics, political science, computer science, biology, etc.
	1078	Calculate the reference angle from an angle in standard position. Determine coterminal angles.
	1080	Estimate or calculate the margin of error; determine the size of the sample necessary for a desired margin of error.